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# Informatively optimal combining, expanding, and establishing traceability in evaluating measurement uncertainties

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## Abbreviations

ANSI – American National Standards Institute
AOIJ – Academic Open Internet Journal
BIPM – International Bureau of Weights and Measures (Bureau international des poids et measures)
CITAC – Cooperation on International Traceability in Analytical Chemistry
CIPM – International Committee for Weights and Measures
CMC – Calibration and measurement capability
DFC – Dimensional factor of confidence
GUM – The Guide to the Expression of Uncertainty in Measurement
EA – European Cooperation for Accreditation
EQCA – Ensuring quality in controlling measurement accuracy
ERAC – Economic and risk analysis criteria
EURACHEM – A network of organizations in Europe, having the objective of establishing a system for the international traceability of chemical measurements and the promotion of good quality practices
IAS – International Accreditation Service
IC – An instrument undergoing calibration or testing
INPL – The National Physical Laboratory of Israel
JCGM – Joint Committie for Guides in Metrology
JLC – Justified level of confidence
LC – Level of confidence
MRA – Mutual Recognition Arrangement
NCSL – National Conference of Standards Laboratories
NMI – National Metrology Institute
OIML – International Organization of Legal Metrology
PIC – Principle of information cyclicity
PIR – Permissible information redundancy
PNR – Perfect numerical ratio
RAO – The Russian Author's Society
RS – Reference standard
QCP – Quality control procedures
SOP – Standard operational procedures
TUR – Tolerance uncertainty ratio
UACS – Universal accuracy classification scale
UTR – Uncertainty tolerance ratio

## Preface

*Study Background*

This mainly theoretical work focuses on a part of fundamental metrological problems that can be demonstratively solved through the new philosophy of justified accuracy estimations, which is being based on qualimetry and Shannon's information theory. Along with informatively optimal measurement uncertainty evaluation that is the major topic in the present study, the list of such problems comprises proper test uncertainty ratios, optimal measurement traceability chains and accuracy classifications, and some more, as well as concerns others beyond metrology directly.

My attempts of applying principles of qualimetry for metrological estimations have been reflected in the series of publications in "Measurement Techniques", starting with the article [1]. Later this approach was improved by using elements of information theory. This possibility occurred owing to the discovery of the *principle of information cyclicity* (PIC) that had been registered by The Russian Author's Society (RAO) [2]. The principle allows optimizing the information concerning quality of infinite variety of objects, including those connected with measurement [3].

Later on the interrelation of informational optimality with mathematical harmony and balance has been revealed, which jointly form the set of conceptions of classification perfection in their dimensional interpretation. In applied metrology the use of such conceptions is not just reasonable but obligatory, because there technical and methodological decisions are always of systemic character and classification nature.

The fusion of some simple methods of qualimetry and information theory, as well as above mentioned conceptions of perfection proved successful theoretically and, expectedly, in practice. Some achievements of this approach in application to metrological and other scientific and technical problems have been reflected in a number of my articles in "OIML Bulletin", and in other journals and proceedings of the international scientific conferences; on some of them this Report has references.

*Summary*

Updated criteria, based on the concept of informational optimality, are proposed as a part of methodological instrument for analyzing modeling functions of measurement, using relative weights of measurement uncertainty contributions. The criteria allow determining informatively sufficient contributions to combined uncertainty and the optimal level of confidence for expanded uncertainty, as well as the test uncertainty ratios and uncertainty tolerance ratios. Commonly used heuristic standard numerical rates in determining corresponding estimates have been analyzed for establishing to what extent they conform to the proposed new approach and criteria, and for inquiring into their informational essence and practical use. Along with informational optimality other conceptions of dimensional perfection, i.e. mathematical harmony and balance are applied to analyze critical levels of confidence and to substantiate the rationality of obtained coverage factors for expanded uncertainties, as well as for other practical aims. Presented practical examples convincingly illustrate the effectiveness of proposed methods. In order to avoid the overloading of main content, basically the Report comprises final results of calculations. Some theoretical details and supplementary research data that should or could be useful for better perception of the gist of the work are presented in Appendixes.

*"Geometry has two great treasures: one is the Theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel"*

-- *Johannes Kepler*

## Introduction

This work basically bears upon the informational optimization of measurement estimations, and in particular is intended for improving the analysis of calibrations and tests being performed in INPL. Known standard procedures of such an analysis are specified by ISO Guide to the expression of uncertainty (GUM) [4]. A generalization of the principles and criteria offered by the work can depend on the reconciliation with GUM of other methodologies (even substantially different from GUM). This, for instance, involves analytical chemistry [5] and other measurement fields with which the Laboratory deals. One of purposes of the work is the find to what extent known traditionally used heuristic criteria are scientifically sound.

There are three basic problems that are somehow or other arising in the theory and practice of measurement accuracy substantiation, especially in dealing with the modeling functions of measurement and in providing measurement traceability; their common trait is in the absence of rigorously proved criteria used. The first problem is the determination of necessary and sufficient contributions to uncertainty budget; the second one is the choosing of adequate level of confidence (LC) for expanded uncertainty; and the third is the determining the best relation between the uncertainty of object undergoing calibration or test on the one hand and the reference uncertainty - on the other, often called test uncertainty ratio (TUR). The present-day situation is being briefly discussed below in this section. I believe the offered study has resulted in successfully overcoming existing difficulties connected with these problems.

The correct identification of uncertainty contributions to be taken into account is the first crucial task for a proper estimating the measurement uncertainty of test and calibration results, measurement methods validation, and measuring equipment verification. ISO/IEC 17025 [6] fairly requires using "appropriate methods of analysis for taking into account all uncertainty components which are of importance in given situation". However, there were no strictly proved ways of achieving this goal.

Meanwhile, metrological experience and results of uncertainty budgets' analysis (including examples entered in this report) indicate a non-well-founded redundancy of the budgets is typical in various fields of measurement. This, in particular, technically excessively complicates the providing of measurement and thus leads to redundant expenses of calibration and testing laboratories and, naturally, for an economy as a whole. In some cases to avoid this drawback it is recommended to focus on those uncertainty sources that have a magnitude of one-third or more of the largest source; less attention should be spent estimating lesser sources [7]. However, this in principle classification ratio, while being seemingly reasonable in practice, is not yet theoretically substantiated and apparently for this reason is not always used.

As for LC, the commonly used 95% level of confidence is well recognized in the Statistical community and then adopted in metrology for evaluating the expanded uncertainty of measurement as a historical artifact, and not as a strictly substantiated value. The increasing of LC (e.g. 99%) or its decreasing (e.g. 90% or even 68%) one may also meet in practice. In some cases a practical task can predefine a degree of adequacy in choosing certain LC as, say, when are being dealt with so-called risks analysis. There is also an attempt of suggesting this approach in combination with an economic optimization for determining the uncertainty being declared by Labs in regard to the calibration and measurement capability (CMC) [8]. However, for many standard measurement methods, measurement standards and measuring instruments the optimization reducing to such approaches most commonly is non-adequate. This is owing to the universalism in applying of above metrological objects demanding strict accuracy classification. But the approaches can likely be useful as an additional tool if they are not violating the main classification criteria of optimization.

Even when possess application universality, the classification ratio and LC apparently should be informatively well-grounded or, preferably, informatively optimal as an intrinsic characteristics of measurement method used in calibration or testing. In particular, it follows that most commonly used and seeming somewhat elusive the one-third ratio and 95% LC (or any others) should be subjected to informational analysis. The same is true for the so called 4:1 Rule, often recommended [9] for TURs, but unproved and not always obtainable in practice.

In my opinion, such a situation with above metrological conceptions is due to the absence of both the scientifically proved criteria and an optimal accuracy classification in applied metrology. It should be noted the classification approach involves also the problem of optimizing the number of classes including, in particular, number of links in a traceability chain that will be solved too. Later on this problem is designated as the determining of *optimal classification integer* and its permissible deviations from the optimality.

The present work applies elements of qualimetry [10] and information theory, and uses specific informatively significant numerical classification ratios for solving problems mentioned above. As the study would not have been done without the appearance of such an approach on the measurement scene, I shall have to preface the discussion with brief acquainting with its basic conceptions and criteria.

I have divided the content of the Report onto following parts: (a) theory, (b) practical examples illustrating theoretical statements (designated in the text as E1, E2, etc.), and (c) appendixes explaining and detailing some of the statements.

## Conceptions and criteria

Although measurement uncertainty is of random nature, it is not a statistical value, because along with a contribution due to repeated measurements (Type A) its evaluation involves a number of non-statistical components. Thus, while using probabilistic calculation technique, the combined and expanded uncertainty should reasonably be explained on qualimetrical basis rather than on statistical. Importantly also that the common probabilistic idea of uncertainties enables to apply elements of information theory in formulating and substantiating needed quantitative criteria.

Beyond question a selection of uncertainty contributions and an estimation of LC relate to the quality of measurement result and its informational sufficiency. Then principles and criteria of doing this, borrowed from qualimetry and information theory or developed on their base, imply that:

(a) the analysis of a modeling function of measurement involves the identification of uncertainty contributions (from their theoretically infinite variety) according to their influence upon the combined uncertainty which, in turn, characterizes the quality (in terms of accuracy) of measurement;

(b) the quality of measurement associated with measurement accuracy ought to be determined strictly in terms of classification, i.e. each method and procedure of measurement belong to certain accuracy grade, or class (even when such a gradation is not yet documentarily specified or somehow indicated);

(c) each grade, or class of accuracy is bounded regarding the composition of contributions of uncertainty to be taken into account, so that the informational sufficiency of each classification group is characterized by the adequate relation between minimum and maximum contribution [◊].

[◊] Note *The providing of information sufficiency in such a way is the only logically justified possibility to classify measurement accuracy for traceability chains, for measurement standards and measuring instruments, and for many other purposes [11].*

A standard modeling function of measurement is being analogous to a function of modeling a quantitative estimation of quality. The output estimate $y$ of measurand as a function of input estimates $x_1, x_2, ..., x_N$ for $N$ quantities is given by $y = f\ (x_1, x_2, ..., x_N)$. Any integral estimate that represents a function (or a set) of some $N$ contributions, such as a modeling function of measurement, can be subjected to qualimetrical analysis. The analysis is always based on determining influences (weights) of contributions on the certain quality of the object undergoing consideration.

Through weights ($K_j$) each "$j$" contribution may be characterized by specific relative index $\rho_j$ so that $1/\rho_j$ indicates to what extent the weight related to this contribution exceeds the weight related to the contribution possessing 50% confidence to be informatively redundant:

$$\rho_j = K_{\varphi o} / K_j, \tag{1}$$

where $K_{\varphi o}$ is the weight of the lesser by value contribution amongst informatively optimal (necessary and sufficient) number $\varphi_o$ of contributions ($\varphi_o \le N$).

In relation to the maximal weight ($K_{max}$) the theoretically stated index, symbolized as $\rho_o$, may be called *informatively optimal classification ratio*:

$$\rho_o = K_{\varphi o} / K_{max} = 1/2\pi \approx 0.159 \tag{2}$$

This ratio may be qualified as the fundamental informational constant, tightly bound with the mathematical constant $\pi$. Its substantiation is performed through the use of information entropy ($H$) regarding two boundary components with weights $K_{max}(\rho)$ and $K_{\varphi o}(\rho)$, provided they form the complete system, i.e. $K_{max}(\rho) + K_{\varphi o}(\rho) = 1$, as the solution of the following equations system:

$$\begin{cases} \rho_o = \arg\,[\varphi_o(\rho) = 1.5]; & (3) \\ \varphi_o = \exp\,(H) = \exp\,[-K_{max}(\rho)\ln K_{max}(\rho) - K_{\varphi o}(\rho)\ln K_{\varphi o}(\rho)]; & (4) \\ K_{max}(\rho) = 1/(1+\rho); & (5) \\ K_{\varphi o}(\rho) = \rho/(1+\rho), & (6) \end{cases}$$

where $\varphi_o(\rho) = 1.5$ is true for the most uncertain classification situation (50% confidence) about allowing or ignoring the lesser component [12]; $K_{max}(\rho)$, and $K_{\varphi o}(\rho)$ are considered as analogs of probabilities enabling the entropy usage.

Another way of substantiating $\varphi_o$, $K_{\varphi o}$ and $\rho_o$ is the analysis of poly-component system that in general form is briefly dealt with in Appendix 1.

Indexes $\rho_j$ and $\rho_o$ will be demonstrated as being kernel in solving measurement accuracy problems mentioned in the Introduction.

*Perfect numerical ratios*

Since measurement is the process of assigning a number to a physical property, the identification of the properties' relations with some numerical ratios that constitute the system possessing conceptual formal-informational features is worthwhile. These features we can define as those related to conceptions of *dimensional perfection*.

Informational *optimality* expressed (2) by $\rho_o$ is one of three perfect numerical ratios (PNR) related to conceptions of dimensional perfection which will be used as methodological instruments in the present study. Another two conceptions are: mathematical *harmony* $f_o = [(\sqrt{5} - 1)/2]^{-1} \approx 0.618$ (harmonious relation) – fundamental constant, otherwise known as *phi*, and *balance* $\lambda = 0.5$ [◊]. By PNR we will imply the arithmetic ratio either of two parts of a whole or (depending on an objective) of one of parts to their sum being related to certain quality.

◊ Note *Virtually PNRs represent objectively existing numerical ratios as preferable in the evolution of nature and human practice; among them the commonly known is the harmony ratio. Provided that systemic connection between PNRs exists, they may be and will be jointly used as one more tool in solving declared problems.*

Any PNR manifestations in nature and human practice always oscillate between ideal and approximations, i.e. are being characterized by deviations from a classification constant $C_{inf}$ (i.e. $\rho_o$, $f_o$, *or* $\lambda$) that in many cases make their accepting problematic. The problem is successfully solved through the conception of informational optimality itself. The deviation equal to $\pm 0.5\rho_o = \pm 1/4\pi$ multiplied on $C_{inf}$ can reasonably be attributed to the constant. This permissible deviation amounts to approximately ±8% of $C_{inf}$. The rigorous substantiation of that is presented in Appendix 2.

While the harmony ($f_o$) is the widely known conception, until now manifestations of balance ($\lambda$) practically are not being draw attention. At the same time, both conceptions are fundamentally associated via Fibonacci numbers ratios, which start with pure balance, and very quickly approach the harmony. This feature is especially noteworthy, some details of which are reflected in the discussion section too. Interestingly, the next after $\lambda = 0.5$ ratio of adjacent numbers in Fibonacci succession, which among other ratios is being characterized by maximum deviation from $f_o$, equals 0.667 that meets the boundary permissible requirement, i.e. $f_o$ $(1 + 1/4\pi)$. Besides, in terms of $C_{inf}$ permissible deviations there is the deep informatively optimal connection via $\rho_o$ between the existing Fibonacci ratios, i.e. $r_\lambda = \lambda\ (1 \pm 1/4\pi)$ and $r_{fo} = f_o\ (1 \pm 1/4\pi)$. The proof consists in determining ratios $r_\lambda = 0.52$ and $r_{fo} = 0.614$ as the solution of following equation: $(f_o - r_\lambda)/f_o = (r_{fo} - \lambda)/r_{fo} = \rho_o$.

Each PNR bears upon simplest case of dividing a whole onto two complementary parts or components that in terms of classification is called *classification dichotomy*. The question, whether information optimality, mathematical harmony and balance represent the complete set of dimensional perfection characteristics implies two approaches: firstly the informational one (informational non-redundancy) and secondly the geometrical one (completeness in various coordinates measures). The dimensional separability of the characteristics is their quality that allows reliably distinguishing between them, and also needs verification. These problems of systemic character are being discussed and solved in Appendix 3.

The found of informatively optimal connection between critical Fibonacci ratios, as well as above considered the systemic character of PNRs classification dichotomy enable to hypothesize that the harmony and balance rank directly among informational conceptions in terms of Shannon's theory. The verification of this hypothesis is dealt with in Appendix 4. To exemplify PNR manifestations jointly among infinite variety in nature we have chosen critical temperature points of chemical elements as an example in inorganic world (Appendix 5), and parameters of blood – in organic world (Appendix 6).

Now, taking into account the discussed basic conceptions and criteria generally, we will proceed to the main content of the present study demonstrating how PNRs allow to solve accuracy problems in metrology.

## Selection of uncertainty contributions

The study is not aimed at creating the comprehensive methodology. For consideration simplicity we presume here uncorrelated input quantities in a model of measurement. In case of correlated ones the approach is essentially the same, but it needs taking into account the correlations calculated as specified in a number of guides, e.g. [4, 13].

When identifying the quality of measurement with properly estimated measurement uncertainty, the variances $u_1^2$, $u_2^2$, ..., $u_N^2$ of contributing uncertainties of input estimates, associated with the output estimate, may be considered as importance measures for the evaluation of uncertainty. These measures represent weights that in the normalized form (when their sum = 1) are being determined as follows:

$$K_j = u_j^2 / \sum_{j=1}^{N} u_j^2 \qquad (7)$$

Then using formulas (1) and (2) and on the base of apparent quantitative classification criterion the selection of informative uncertainty components is carried out by the condition $\rho_j \geq \rho_o$ or as:

$$u_j^2 \geq u_{max}^2 / 2\pi \tag{8}$$

It follows from such a selection that the number $m = (N - \varphi_o)$ of informatively redundant uncertainty contributions (conditioning $K_j < K_{\varphi o}$) is being excluded from calculating the combined uncertainty. As any optimization the selection results in certain losses of quality compared with using all $N$ components [◊]. By definition these losses ($L_q$) are being also optimal.

[◊] Note *The use for the estimation of all components, including informatively redundant ones, is related to something that one can call 'absolute quality' that is meaningless for the classification approach we are dealing with, and therefore is out of real needs.*

Losses $L_q$ can be expressed in form of the relative error of selection as follows [14]:

$$L_q = F\sum_{i=1}^{m} K_i = (1/NK_{max})\sum_{i=1}^{m} K_i = (1/Nu_{max}^2)\sum_{i=1}^{m} u_i^2 \ , \tag{9}$$

where: $F = 1/NK_{max}$ is the form-factor of weights diagram,
$i$ = the symbol for $m$ redundant contributions of measurement uncertainty.

Being subsidiary relative characteristic, $L_q$ can be rather important and applicable for various comparative analyses connected with measurement uncertainty estimations. One of usages in question is presented in this work when discussing levels of confidence for expanded uncertainty evaluation.

The determination of optimal quality losses is also of practical concern when comparing the ways of calculating a combined uncertainty. This together with components selection is illustrated on various examples (E1).

The important feature of expression (9) also is its practical applicability to the widest variety of weights' diagrams. This is owing to (a) that $L_q$ as the estimation error due to diagrams' difference never exceeds $\rho_o$, and (b) that the linear diagram of weights may reasonably be accepted as being the versatile model, for which the error ranges within 0.013 ÷ 0.05. The proof of this statement is briefly presented in Appendix 7.

## Critical and optimal levels of confidence

Dimensionally it is convenient to characterize an interval having certain percentage level of confidence by the following relative value:

$$\mathcal{E}_{LC} = 1/(1 - \text{LC}/100), \tag{10}$$

which we name *dimensional factor of confidence* (DFC). There is one chance from the number equal to the factor that the value of the measurand lies outside the interval.

It follows from the informational approach that $\mathcal{E}_{50} = 1/(1 - 50/100) = 2$, being the reference value, corresponds with 50% confidence. This value is of principle significance for determining the optimal level of confidence $\text{LC}_{jo}$ for each component in a system of $N \geq 2$ components. In terms of informational optimality, that is when considering the ratio $\mathcal{E}_{50}/\mathcal{E}_{LCjo} = \rho_{jo}$, the further determining is carried out by means of the following equations system:

$$\begin{cases} 1 / (1 - LC_{jo} / 100) = \mathcal{E}_{LCjo} & (11) \\ \mathcal{E}_{50}/\mathcal{E}_{LCjo} = \rho_{jo} & (12) \end{cases}$$

The solution of the system regarding $LC_{jo}$ (when taking into account the above factor $\mathcal{E}_{50} = 2$) results in the general expression (13), as well as singly through uncertainty contributions, i.e. $LC_o(u_j)$, to the expression (14):

$$LC_{jo} = (1 - \rho_{jo} / \mathcal{E}_{50})*100\% = (1 - 0.5\, \rho_{jo})*100\% = = (1 - 0.5\, K_{\varphi o}/K_j)*100\%, \quad (13)$$

$$LC_o(u_j) = (1 - 0.08\, u_{max}^2 / u_j^2)*100\% \quad (14)$$

In case $K_{max}$ = constant, for a non-redundant practical use the range of $LC_{jo}$ is from 50% ($K_j = K_{\varphi o}$) up to 92% ($K_j = K_{max}$). This range characterizes informatively necessary and sufficient confidence to the estimation quality. The increasing of the quality above this optimum on account of information redundancy is limited by 96% level of confidence for the contribution with maximal weight. The decreasing of the quality reaches 0% of the level for $K_j$ when $K_j = 0.5K_{\varphi o}$. A further redundancy increase ($K_j < 0.5K_{\varphi o}$) leads to the failure of classification criterion and formally results in a negative confidence level for at least one of contributing components. Thus, when applying formula (14), the most concrete magnitude for the common LC is 92%. Because LC = 95% is located within permissible redundancy, this confirms the practical possibility of its usage and will be singly treated further.

Fragmentarily the obtained results are illustrated on Fig. 1 as the interrelation $LC_j = 100[(96 - LC_\varphi)/(100 - LC_\varphi)]$ between the current confidence ($LC_j$) on the one hand and the classification-minimal confidence ($LC_\varphi$) on the other hand in the system of two critical classification components.

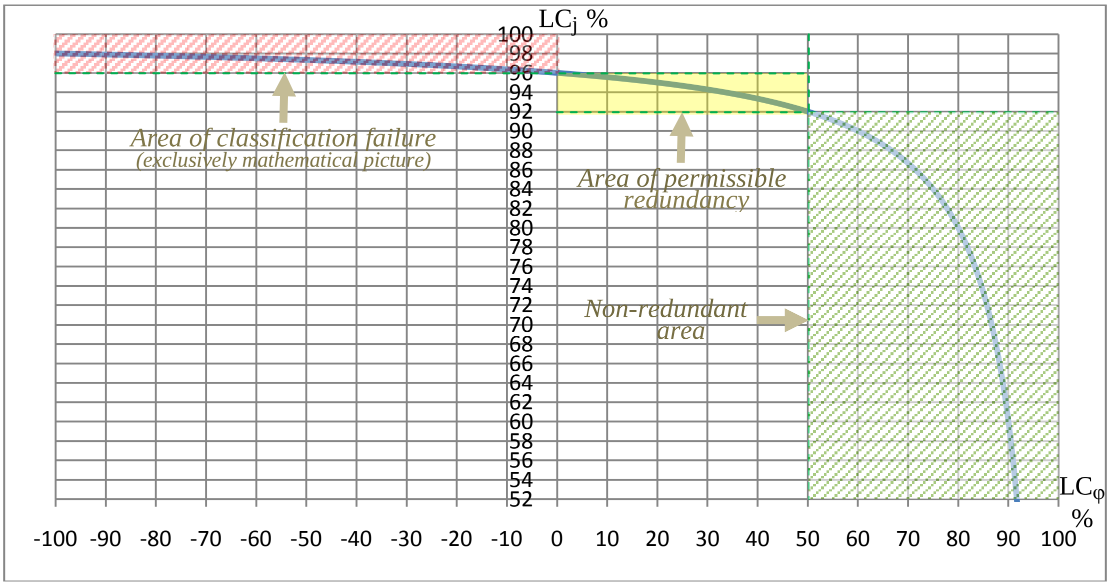


Fig.1: Functional interconnection between $LC_j$ and $LC_\varphi$

We will analyze the obtained critical LC and DFC, to what extent they meet requirements of dimensional perfection expressed by PNR. Calculations of dimensional factors' ratios of confidence regarding above critical levels of confidence (including 95%) result in the data presented in Table 1.

Table 1: Results of analyzing relations of DFC for the critical LCs

| LC (%) | $\varepsilon_{LOC}$ | $\varepsilon_{50}/\varepsilon_{92}$ | $\varepsilon_{92}/\varepsilon_{95}$ | $\varepsilon_{92}/\varepsilon_{96}$ | Identification to $C_{inf}$ | Deviation from the constant (%) |
|---|---|---|---|---|---|---|
| 50 | 2 | | | | | |
| 92 | 12.5 | 0.16 | | | $\rho_o = 0.159$ | 0.6 |
| 95 | 20 | | 0.625 | | $f_o = 0.618$ | 1.1 |
| 96 | 25 | | | 0.5 | $\lambda = 0.5$ | 0 |

Table 1 convincingly demonstrates the informational perfection (in terms of PNRs) of the system composed by the critical levels of confidence being determined according to the proposed function (13). The additional useful argumentation in favor of formula (13) will be reflected in the further consideration when analyzing adequate critical coverage factors. The practical illustration is given (E1).

### Classification ratios and integers. Traceability characteristics

Now we will focus on the already mentioned numerical criteria: $u_j/u_{max} = 1/3$, TUR = 4:1, and (in the separate section) LC = 95% as estimates rates, having been most often used in metrological practice. A priori one can hypothesize that since these rates are result of persistent practice, they should be more or less adequate to proposed classification requirements, i.e. in certain degree possess or near informational optimality. This hypothesis relies on the objective existence of evolutionary perfection in nature and human activity and will be examined farther.

*One-third ratio. Classification integers*

Ratio $u_j/u_{max} = 1/3$ represents square root of variances ratio $u_j^2/u_{max}^2 = 0.111$, thus, if being used as a classification ratio, it is located in the area of *permissible information redundancy* (PIR), that is within the range from $1/4\pi$ to $1/2\pi$. Interestingly, in terms of informatively optimal levels of confidence (13) this variances ratio conforms to $LC_o$ = 94.5%, i.e. relatively close to commonly used 95%.

In order to find out whether one-third ratio meets PNR requirements we will consider the inverse value $u_{max}/u_j = 3$. This integer ($X$) is adequate to often used tolerance uncertainty ratio 3:1. In metrological practice if taking also into account TURs, integers $X$ = 1, 2, 4, 5, 10, and even 20 are used too [9, 18, 19, 20].

The degree of classification optimality of an integer within rounding interval can be determined in regard to PNRs. For a normalized system of two components $1/x$ and $(1 - 1/x)$, where $x \geq 1$, the optimal classification integer can be found using the absolute value of difference between the integer and $(1/x)/(1 - 1/x)$, as the rounding off ($R$) values $X_{o(C_{inf})} = R\{x\}$, i.e.: $X_{o(\rho)} = R\{arg\ min\ |\rho_o - (1/x)/(1 - 1/x)| = 0\}$;

$X_{o(f)} = R\{arg\ min\ |f_o – (1/x)/(1 – 1/x)| = 0\}$; $X_{o(\lambda)} = R\{arg\ min\ |\lambda – (1/x)/(1 – 1/x)| = 0\}$.

Graphically the calculated outcomes are presented in Fig. 2.

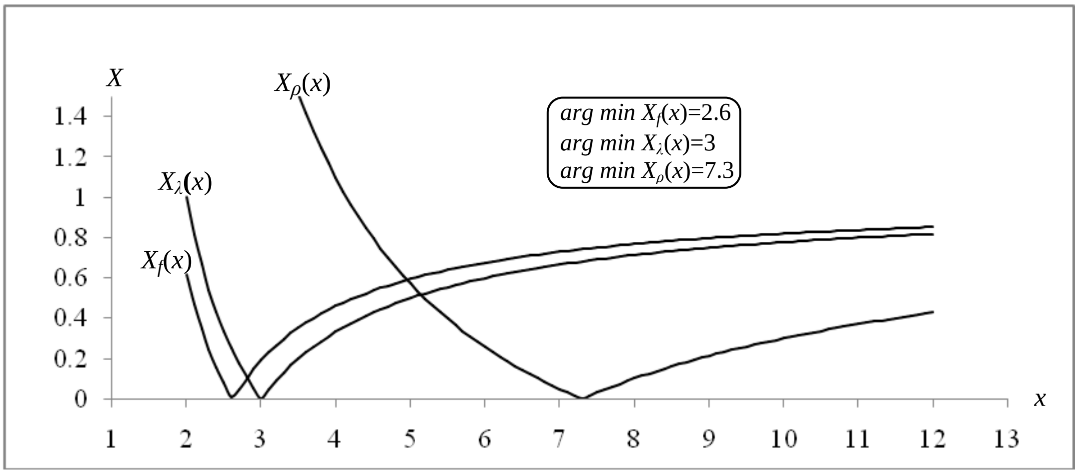


Fig. 2: The optimality of integer 7, and the harmony and balance of integer 3

The coincidence of $X_{o(f)} = X_{o(\lambda)} = X_{o(f,\lambda)} = 3$ indicates on the existence of just one boundary classification integer 3 that meets the requirements of both harmony and balance. Thus in a quantitative classification the integer 3 presumably represents the minimum permissible value for the integrity of the system (consisting of more than two components) in terms of both harmony and balance, unlike classification dichotomy where PNRs are possible just singly from each other.

The highly important result obtained is $X_{o(\rho)} = 7$ which points onto the unique property of the well-known ‘magical seven’ as the optimal classification integer that, in turn, will be used further in establishing optimal traceability chains. More detailed discussion, including optimality's boundaries (the so-called “7±2 phenomenon”), are dealt with in Appendix 8.

*Test uncertainty ratio*

There is significant gap in understanding the measurement traceability: the absence of sound quantitative criterion for this important characteristic of measurement. The determination of TUR between the object undergoing calibration or test and the reference one, performed in the process of measurement uncertainty evaluation, is obviously the way to fill up the gap. Since the evaluation deals with the budget of uncertainty that embraces a calibration or test procedure in full, in this case the criticism of TUR as not taking into account of “critical factors that contribute to a measurement uncertainty from the device under test” [18] becomes bankrupt.

Unlike using classification integers, the providing traceability with TUR in many practical cases requires more accurate fractional numerical criterion. This is easily achievable by using the proposed above principle of LC optimization to determining the informatively sufficient tolerance uncertainty interval: from $TUR_{min} = \sqrt{2\pi} = 2.5$ up to $TUR_{max} = \sqrt{4\pi} = 3.5$ for 92% and 96% of the confidence $LC_{TUR}$ of TUR estimation respectively (that is illustrated by Fig. 3). The medium value within the interval is $TUR_{med} = 0.5(TUR_{min} + TUR_{max}) = 3$ (that is the above mentioned classification integer). Clearly, in such a consideration 4:1 Rule represents the rounding off value of $TUR_{max}$ and possesses some extra redundancy.

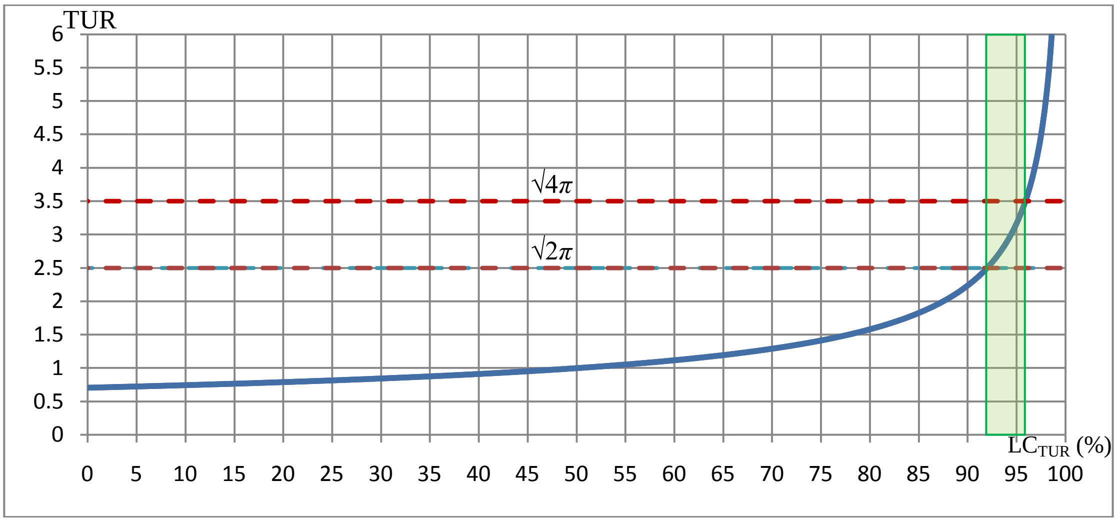


Fig. 3: The dependence of TUR from estimation confidence percentage

In terms of quality loss the traceability can be characterized by the conversion of formula (9) for two components as the following expression for *traceability quality loss* ($L_{qt}$): $L_{qt} = 0.5/(\text{TUR})^2$; and the informatively sufficient $L_{qt}$ interval is determined as $L_{qt(min)} = 0.5/(\text{TUR}_{max})^2 = 0.04$, and $L_{qt(max)} = 0.5/(\text{TUR}_{min})^2 = 0.08$.

In some cases especially for a highest level of accuracy intrinsic to national metrology institutes (NMI) the traceability even does not satisfy $TUR_{min}$ and $L_{qt(max)}$. This, for instance, is the characteristic for TUR between mechanical and interferometric measurement of gauge blocks (from 0.5mm up to 100mm nominal). Based on the INPL's declaration in this field, the respective TUR is between 1.7 and 2.0. Results of analogous examining of NMIs are presented (E2) too.

It should be noted, there are sometimes attempts of substantiating traceability in a practice of calibration and test over results of interlaboratory comparisons, and not by means of test uncertainty ratios. The criticism of and the proved failure from such an approach one can find in [19] [◊].

◊ Note *Prof. Paul. De Bievre in [19] draws the following conclusion: "Interlaboratory comparisons do not provide traceability of values of measurement results because only deliver information a posteriory. Therefore, they cannot establish traceability. The establishment of traceability of measurement result is a task for every single measurement laboratory on its own and does require knowledge a priory. Interlaboratory comparisons of the results of different laboratories are an a posteriory process. They yield another useful product: the establishment of the degree of reproducibility of results of different laboratories, or degree of equivalence between the measurement capability of the participating laboratories".*

*Traceability chain*

A traceability chain represents the accuracy hierarchy where adjacent levels are related through TURs. The problem of optimal classification integer $\tau_o$ as an informatively sufficient number $\tau$ of levels and their classification periodicity in such hierarchies is also of interest for metrological estimations. Over the averaged out estimation the periodicity is achievable to certain number ($\tau$) of multiplied by itself ratios TURmed/TURmin and TURmax/TURmed. Since a traceability chain is characterized by the number $\tau$ of links between ($\tau$ + 1) of elements, the following equations are true within the range of minimum and maximum test uncertainty ratios:

$$\mathrm{TURmed}^{\,\tau} = \mathrm{TURmin}^{\,\tau+1}, \quad \text{and } \mathrm{TURmax}^{\,\tau} = \mathrm{TURmed}^{\,\tau+1}$$

There are two optimums for the estimation error $\delta_{\mathrm{TUR}}(\tau)$ intrinsic to these equations:

$$\tau_{o1} = arg\ min\ |\delta_{\mathrm{TUR1}}(\tau)| = arg\ min\ |\mathrm{TUR}_{\mathrm{med}}^{\ 1/\tau} - \mathrm{TUR}_{\mathrm{max}}/\mathrm{TUR}_{\mathrm{med}}|$$
$$\tau_{o2} = arg\ min\ |\delta_{\mathrm{TUR2}}(\tau)| = arg\ min\ |\mathrm{TUR}_{\mathrm{med}}^{\ 1/\tau} - \mathrm{TUR}_{\mathrm{med}}/\mathrm{TUR}_{\mathrm{min}}|$$

Graphically the results of calculations are presented in Fig. 4, which show in fact the availability of two integers of optimum: $\tau_{o1}$ = 6 and $\tau_{o2}$ = 7 that, corresponding to the above $X_o(\rho)$ = 7, meets the requirements of Miller's 7±2 phenomenon. With the aid of PNRs, the phenomenon is proved in Appendix 8.

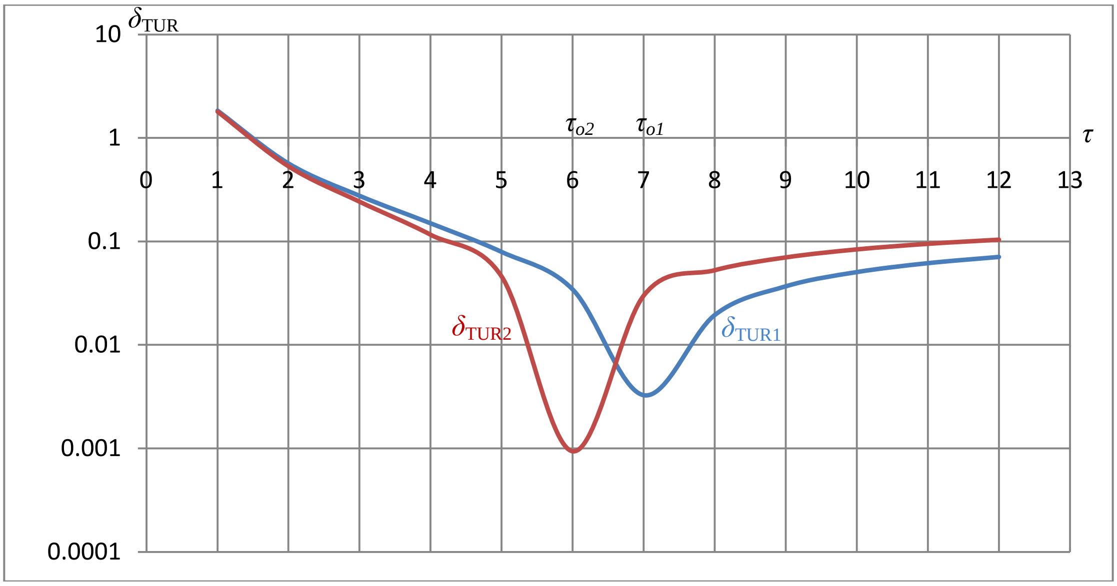


Fig. 4: Informational optimality of 6 and 7 as traceability chain integers

Long ago developed in the fields of length and mass measurements five orders of calibration accuracy of end standards [20] and nine classes of accuracy of weights standards [21] is one of illustrations of 7±2 manifestation in metrology. At the other end, boundary values in both cases reveal the hypothetical possibility of the systems' further improving. An imperfection of classification system as a deviation from ideal classification integer 7 may characterize also a deficiency of the inner structure of system. The above mentioned accuracy classifications is exemplified (E3).

The optimality of integers 6 and 7 represent key structural base of the proposed in [22] Universal Accuracy Classification (Appendix 9), which on each hierarchical level consists of seven relative classification elements forming six subclasses.

### Dimensional perfection of optimal traceability chain

The above considerations' outcome allow to propound the hypothesis that the full dimensional perfection (the optimality, the harmony, and the balance) is adequate to only single integer representing informatively optimal number ($t_o$) of ties in the optimal traceability chain. By optimal traceability chain we will call a system consisting of ($t_o$ + 1) objects of measurement which, starting with of the second object (over accuracy) undergo calibration with the optimal TUR for each one. Thus, supposedly $t_o = \tau_{o1} = 6$. If such supposition is correct, the value determined as an optimal TUR raised to the sixth power may be called an optimal traceability index of optimal traceability chain. For checking up the hypothesis in terms of dimensional perfection, one can look upon the functions representing the following ratios:

$$p_1(t) = \mathrm{TUR}_{\min}^{t}/(\mathrm{TUR}_{\max}^{t} - \mathrm{TUR}_{\min}^{t}),\quad p_2(t) = \mathrm{TUR}_{\min}^{t}/(\mathrm{TUR}_{\mathrm{med}}^{t} - \mathrm{TUR}_{\min}^{t}),$$
$$\text{and}\quad p_3(t) = \mathrm{TUR}_{\mathrm{med}}^{t}/(\mathrm{TUR}_{\max}^{t} - \mathrm{TUR}_{\mathrm{med}}^{t})$$

The functions have been analyzed (Fig. 5) by the following condition:

$$t_o = R\{arg\ min[|p_1(t) - \rho_o|]\} = R\{arg\ min[|p_2(t) - \lambda|]\} = R\{arg\ min[|p_3(t) - f_o|]\} = 6 \quad (15)$$

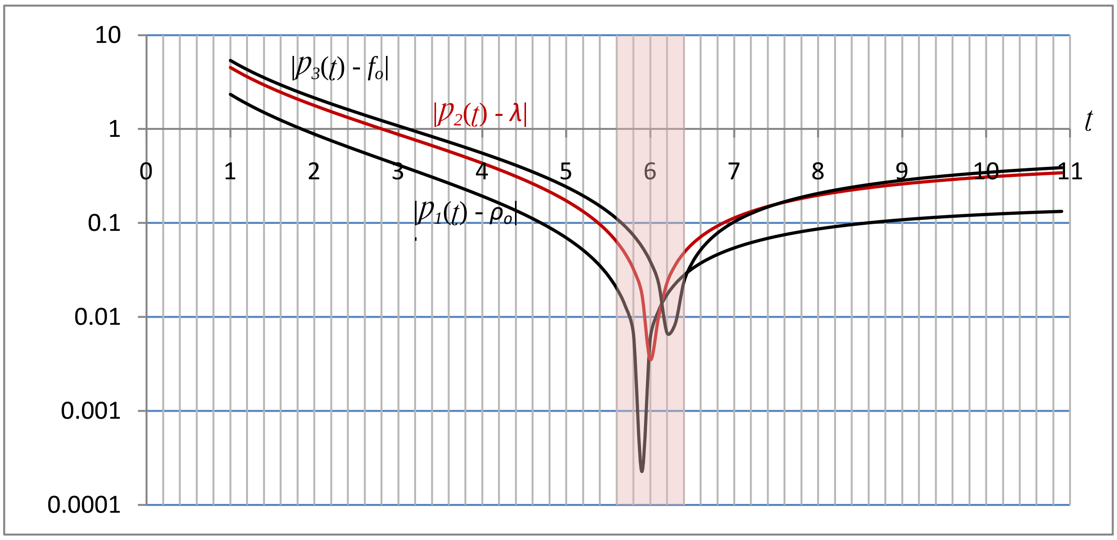


Fig. 5: Graphical illustration of informational optimality and dimensional perfection of integer 6 considered here as a number of ties in an optimal traceability chain. The area of a rounding off for the integer is shown painted

Another condition of acknowledging the hypothesis: the deviation of $p_1(t_o)$, $p_2(t_o)$, and $p_3(t_o)$ from respective $C_{inf}$ is to be no more than 8%. Table 2 demonstrates the satisfaction of this requirement.

Table 2: The adequacy of ratios $p_1(t_o)$, $p_2(t_o)$, and $p_3(t_o)$ to information constants

| Ratios $p_1(t_o)$, $p_2(t_o)$, and $p_3(t_o)$ expressions | Ratio result | Identification to $C_{inf}$ | Deviation from $C_{inf}$ (%) |
|---|---|---|---|
| $(\mathrm{TUR}_{\min})^6/[(\mathrm{TUR}_{\max})^6-(\mathrm{TUR}_{\min})^6]$ | 0.153 | $\rho_o$ | 3.8 |
| $(\mathrm{TUR}_{\min})^6/[(\mathrm{TUR}_{\mathrm{med}})^6-(\mathrm{TUR}_{\min})^6]$ | 0.503 | $\lambda$ | -0.6 |
| $(\mathrm{TUR}_{\mathrm{med}})^6/[(\mathrm{TUR}_{\max})^6-(\mathrm{TUR}_{\mathrm{med}})^6]$ | 0.657 | $f_o$ | -6 |

## Peculiarity of 95% level of confidence

The determination of *informatively optimal* $LC_{opt}$ in the range of *permissible information redundancy* (PIR) can be carried out by using Benford's Law [23]. Since PIR ranges between $1/4\pi$ and $1/2\pi$, these boundaries correspond to the integers of classification character $R\{2\pi\} = 6$ and $R\{4\pi\} = 13$. The critical integer $z_o = R\{1/\rho_{or}\}$ of optimal redundancy can be determined by using Benford's probabilities.

The method of investigation based on the Benford's Law consists in the following. Any integer $z$ greater than one can be used as the base of certain system of numbers; the system will employ $z$ different digits. Benford's probability $P(d)$ of any number $d$ from 1 to $(z - 1)$ is calculated as follows:

$$P(d) = log_z (1 + 1/d) \tag{16}$$

These probabilities form a complete group of independent events, i.e. their sum = 1, and a logarithmic sequence has obvious classification character. In so doing, the informatively sufficient number:

$$d_{\varphi o} = 1/\rho_{or} = \exp [-\sum_{i=1}^{(z-1)} P(d)_i \ ln\ P(d)], \tag{17}$$

and thus for $z = R\{4\pi\} = 13$ the critical integer $z_o$ is calculated as follows:

$$z_o = (R\{d_{\varphi o}\}+1) = 1 - R\{\exp \sum_{i=1}^{(z-1)} log_{13}[1+1/d]*ln\ (log_{13}[1+1/d])\} = 10 \tag{18}$$

Incidentally, this result is of great original significance because it proves the informational optimality of decimal numbering system.

Another way of achieving this result [24] consists in applying the minimum $P_{min} = P\ [d = (z - 1)]$ and maximum $P_{max} = P\ (d = 1)$ Benford's probabilities that form the system of two components. For their ratio the optimization criterion may be expressed as follows:

$$(P_{min} / P_{max})_o = P\ [d = (z - 1)]\ /\ P\ (d = 1) = \rho_o = 1/2\pi \tag{19}$$

The index of optimal classification $z_{o1} = 10$ (that is the logarithm base) is determined as follows:

$$z_{o1} = R\{arg\ min\ |\ log_z\ [1 + 1/(z - 1)]/\ log_z\ (1 + 1) - (1/2\pi)|\} = 10 \tag{20}$$

Now, coming back to expression (13), using results (18) and (20), the optimally redundant level of confidence one may determine as follows:

$$LC_{opt} = [1 - 0.5\ (1/\ z_o)]*100\% = 95\% \tag{21}$$

It is also noteworthy in the systems of components' pairs consisting of $C_{inf}$ and (1-$C_{inf}$) regarding conceptions of harmony and balance the following are outcomes:

$\rho_{fo} = \rho_o f_o = 0.618/2\pi = 0.1$, and respectively $LC_{fo} = (1 – 0.5\, \rho_{fo})*100\% = 95\%$;

$\rho_\lambda = \rho_o \lambda = 0.5/2\pi = 0.08$, and respectively $LC_{fo} = (1 – 0.5\, \rho_\lambda)*100\% = 96\%$.

Therefore, if using LC of the contributing component with $K_{max}$ as the common one for the system, the range 92% ÷ 96% favors the commonly used 95% as:

(a) being informatively permissible, but redundant;
(b) possessing optimal information redundancy.

So the hypothesis of adequacy of commonly used one-third ratio and 95% level of confidence to the informational classification principle is true. It should be noted that LC = 95%, conforming with nearly average of permissible redundancy, in terms of generally accepted rule of providing reliability of informational estimates is even preferable. However, such a preference, while possessing heuristic value, explicitly causes additional estimation uncertainty.

## Influence of levels of confidence on quality loss

A comparative analysis of relative quality losses $L_q$ regarding various LC is of concern. The peculiarity of the analysis consists in the significant dependence of $L_q$ on the form of weights' diagram. The simplest and convenient model is the approximated linear diagram of weights transformed in such a way that the product of transformed maximal weight $K_{mt}$ and number of contributions $N_t$ equals 1. In so doing and taking into account the formula (9), the following final expression is true for such a model:

$$L_{qt} = 0.5\, (\rho_{LOC})^2; \quad (22)$$
$$\rho_{LOCt} = 2(1 – LC /100\%), \quad (23)$$

where $\rho_{LOCt}$ is the classification ratio intrinsic to certain LC, determined from LC = $(1 – 0.5\, \rho_{LOCt})*100\%$. Graphically functions (22), (23) are presented in Fig. 6.

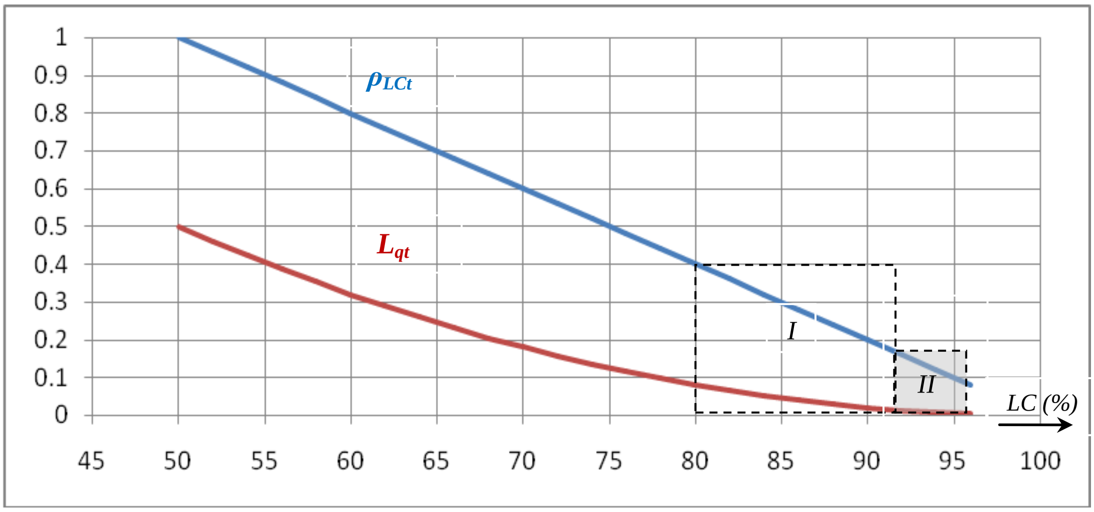


Fig. 6: Graphical illustration $\rho_{LOCt}$ and $L_{qt}$ as functions of LC

This model shows that:

(a) relative to $max$ ($L_{qt}$) = 0.5 the informatively redundant (maximum permissible) quality loss is ($0.5\rho_o$) = 0.08 that corresponds to approximately LC = 80%. Possibly the diapason of LC from 80% to 92% (dotted area $I$ on Fig.5) bounds permissible confidence levels intrinsic to concrete modeling functions of measurement;

(b) in the range of critical LCs from 92% (for $\rho_{LOCt}$ = 0.16) to 96% (for $\rho_{LOCt}$ = 0.08), indicated on Fig. 6 as dotted colored area $II$, the $L_{qt}$ ranges from 1.28% to 0.3% respectively. This demonstrates informationally negligible losses and, therefore, the practical absence of LCs influence within this range.

### Intrinsic and common coverage factors and confidence levels

Clearly, when dealing with an uncertainty budget, the obtained informatively optimal levels of confidence may be jointly used with the statistical instrument of determining coverage factors for calculating valid expanded uncertainty. Normal distribution or Student distribution might be used for this purpose. Theoretical statements and calculations will further be derived based on normal distribution.

The calculation may result in determining either intrinsic coverage factors $k_{in}$ or the common coverage factor $k_c$. The usage of $k_{in}$ is aimed at the precise estimation of expanded uncertainty that is the unique for certain measurement model, whereas $k_c$ is convenient for traditional universalizing the uncertainty expansion. In case of normal distribution the calculations are being performed as follows:

$$k_{in}= ([\sum_{j=1}^{N-m} (k_j u_j)^2]/[\sum_{j=1}^{N-m} u_j^2])^{1/2} \qquad (24)$$

$$k_c = 1.75 \text{ (that resembles } LC_{oj} = 92\% \text{ when } u_j = u_{max}), \qquad (25)$$

where $k_j = \arg\,[\mathrm{erf}\,(k/\sqrt{2}) = LC_{oj}/100] = \arg\,[\mathrm{erf}(k/\sqrt{2}) = (1 - 0.08\, u_{max}^2/u_j^2)]$, (26)
erf ($k/\sqrt{2}$) is the Gauss error function.

In so doing, the intrinsic level of confidence $LC_{in}$ or the common one $LC_c$ = 92% (or either in the range of permissible redundancy, i.e. from 92% to 96%) correspond according to the error function either to $k_{in}$ or $k_c$ respectively.

From the above consideration one may conclude that the feature of applying any $LC_c$ is an incontestable overstating of expanded uncertainty, except the theoretical case when all selected uncertainty contributions are equal. At the same time this traditional way provides a habitual unified estimation norm. Clearly the passing from $LC_c$ to $LC_{in}$ requires changing the philosophy of uncertainty expansion.

### Dimensional perfection of the system of critical coverage factors

In accordance with the proposed approach permissible coverage factors range within boundary critical values $k_{min}$ = 0.67 (for 50% confidence) and $k_{max}$ = 2.05 (for 96% confidence). Another critical value in this range is the optimal coverage factor $k_o$ = 1.75 (for 92% confidence) that divides the range onto two parts so that the sub-range from $k_o$ to $k_{max}$ is being characterized by permissible redundancy. We will analyze these results, to what extent they meet requirements of dimensional perfection expressed by perfect numerical ratios. The results of analysis that prove the dimensional perfection are presented in Table 3.

Table 3: Data proving the dimensional perfection of permissible coverage factors

| # | Name of relative value ($V_r$) | $V_r$ expression | $V_r$ numerical | $C_{inf}$ | Deviation $V_r$ from $C_{inf}$ |
|---|---|---|---|---|---|
| 1 | Maximum range of permissible redundancy | $(k_{max} - k_o)/k_o$ | 0.171 | $\rho_o$ | -7.8% |
| 2 | Mean range of permissible redundancy | $2(k_{max} - k_o)/(k_{max}+ k_o)$ | 0.158 | $\rho_o$ | -0.6% |
| 3 | Minimum range of permissible redundancy | $(k_{max} - k_o)/k_{max}$ | 0.146 | $\rho_o$ | +8% |
| 4 | Minimum range of permissible sufficiency | $k_{min}/( k_o - k_{min})$ | 0.620 | $f_o$ | +0.3% |
| 5 | Maximum range of permissible sufficiency | $k_{min}/( k_{max}– k_{min})$ | 0.486 | $\lambda$ | -2.8% |

Over obtained permissible coverage factors Table 3 demonstrates:

(a) the nearly ideal correspondence between the range (from 92% to 96%) of $LC_c$ that characterizes necessary and sufficient information on the accuracy of measurement on the one hand, and of boundaries of permissible deviation of relative values of coverage factor that match this range and meet the requirement of permissible (±8%) deviations for $\rho_o$ on the other hand;

(b) the very good adequacy of minimum and maximum range of permissible sufficiency (from 50% to 92%, and from 50% to 96% LC correspondingly) to the harmony and balance respectively.

This outcome allows stating the LC determination by formula (14) is definitely adequate to conceptions of dimensional perfection, and this is the significant argument in favor of its practical usage.

Table 3 indicates also onto the existence of interrelation of PNRs. This is of concern both to understanding PNRs themselves and as being significant also in aspects of measurement; more about the interrelation is presented in Appendix 3.

Interestingly, when considering relations between $k_\sigma = 1$ (for 68% confidence), related to one standard deviation being the critical parameter of normal distribution, and the above critical coverage factors, the conditions of dimensional perfection are also satisfied (see Table 4).

Table 4: Relation of $k_\sigma$ with $k_{min}$ , $k_{max}$ and $k_o$

| $V_r$ expression | $V_r$ numerical | $C_{inf}$ | Deviation $V_r$ from $C_{inf}$ |
|---|---|---|---|
| $k_{min}/k_\sigma$ | 0.67 | $f_o$ | +8% |
| $k_\sigma/k_o$ | 0.571 | $f_o$ | -7.6% |
| $k_\sigma/k_{max}$ | 0.488 | $\lambda$ | -2.4% |

Data of Table 4 show the surprising existence of harmony and balance between the above parameters of normal distribution.

**Resolution of traceability problem in regard to test uncertainty ratios and of adequate CMC**

Along with the uncertainty budget the measurement traceability is a separate significant factor which should be taken into account in establishing CMC that are being declared by calibration and some testing laboratories. Quite often this rule is being violated. Further discussion opens possible solving the problem. The proposed procedure is illustrated by practical example (E4).

Scarce traceability can lead to perceptible mistake due to insufficient TUR, namely to the overestimation of the declared CMC uncertainty. Clearly, there are alternative ways of correcting the situation and achieving an acceptable TUR: either 1) by technical measures aimed at lowering the uncertainty of reference standard (or by replacing with the new one of higher accuracy), or 2) by increasing the uncertainty of the object undergoing calibration. In many cases the second way is more practicable. Obviously both ways require substantiated quantitative criterion for acceptable, preferably optimal traceability, and practical procedure of CMC correction.

Above established the informatively sufficient interval between $TUR_{min}$ = 2.5 and $TUR_{max}$ = 3.5 one should consider as just a basic numerical criterion of optimal measurement traceability. The practical application of this criterion by using the existing way of summarizing uncertainty components may result in a deflection from adequate informational optimality, i.e. to inaccurate estimation.

The important point to be noted is simply that there is almost no a priori connection between relatively independent uncertainty budgets in regard to the optimality of expanded uncertainties of the reference standard (*RS*) and of the instrument undergoing calibration or testing (*IC*). To achieve the adequacy the uncertainties evaluation ought to base on the intrinsic levels of confidence and intrinsic coverage factors for both *RS* and *IC*. Towards this end the analysis shall involve the following procedure:

1. In accordance with (8) the selection of informative uncertainty components:

$$u_{j(RS)}^2 \geq u_{max(RS)}^2 /2\pi, \qquad (27)$$
$$u_{j(IC)}^2 \geq u_{max(IC)}^2 /2\pi. \qquad (28)$$

2. In accordance with (14) the calculation of informatively optimal level of confidence for each informative component:

$$\mathrm{LC_o}(u_{j(RS)}) = (1– 0.08\, u_{max(RS)}^2/ u_{j(RS)}^2)*100\%, \qquad (29)$$
$$\mathrm{LC_o}(u_{j(IC)}) = (1– 0.08\, u_{max(IC)}^2/ u_{j(IC)}^2)*100\%. \qquad (30)$$

3. In accordance with (26) the determination of informatively optimal coverage factors for each informative component:

$$k_{j(RS)} = \arg\,[\mathrm{erf}(k /\surd 2) = \mathrm{LC_o}(u_{j(RS)})], \qquad (31)$$
$$k_{j(IC)} = \arg\,[\mathrm{erf}(k /\surd 2) = \mathrm{LC_o}(u_{j(IC)})]. \qquad (32)$$

4. In accordance with (24) the calculation of intrinsic coverage factors for $(N - m)$ and $(N^* - m^*)$ informative components of *RS* and *IC* respectively:

$$k_{in(RS)} = \left(\left[\sum_{j=1}^{N-m} (k_{j(RS)}u_{j(RS)})^2\right] / \left[\sum_{j=1}^{N-m} u_{j(RS)}^2\right]\right)^{1/2}, \quad (33)$$

$$k_{in(IC)} = \left(\left[\sum_{j=1}^{N^*-m^*} (k_{j(IC)}u_{j(IC)})^2\right] / \left[\sum_{j=1}^{N^*-m^*} u_{j(IC)}^2\right]\right)^{1/2}, \quad (34)$$

where: $\left[\sum_{j=1}^{N-m} u_{j(RS)}^2\right]^{1/2} = u_{c(RS)}$, and $\left[\sum_{j=1}^{N^*-m^*} u_{j(IC)}^2\right]^{1/2} = u_{c(IC)}$ are combined uncertainties.

5. Establishing the adequacy between combined uncertainties by the like coverage factor:

$$u_{c(RS)ad} = u_{c(RS)}[k_{in(IC)} / k_{in(RS)}] \quad (35)$$

6. Determining the real test uncertainty ratio in the case:

$$\mathrm{TUR_r} = u_{c(IC)} / u_{c(RS)ad} \quad (36)$$

7. Checking the informational sufficiency by the condition: $\mathrm{TUR_r} \geq 2.5$. If this condition is not satisfied, likely the combined uncertainty for *IC* should be accordingly increased. Clearly this leads to some lowering the CMC uncertainty in regard to certain calibration or testing activity.

**Quantitative criterion for MRA verification**

The main objectives of the CIPM-MRA are known to establish the degree of equivalence of the national measurement standard to ensure world-wide uniformity of measurement and to provide for the mutual recognition of calibration and measurement certificates issued by the NMIs [25]. Unfortunately, the quantitative notion of the *degree* and the *equivalence* for this instance has not been defined.

Basing on the proposed approach, and by introducing the reasonable in this case notion of *equivalence over accuracy classification* or (in a more generalized sense) *over accuracy levels*, it becomes possible to fill up this gap. The problem is being simply reduced to the routine analysis of CMCs of MMIs participating in MRA according to hierarchical levels of accuracy. This is being performed by means of comparing CMC of each "*i*" NMI with the best in the pull CMC, i.e. by comparing the uncertainties $U_i$ with the reference uncertainty $U_r$ respectively.

The comparison is to be carried out by the following criterion:

$$U_i \leq U_r\sqrt{2\pi} \quad (37)$$

Unsatisfying to this condition NMIs should be considered as being belonged to other classification group and, therefore as MRA outliers. Examples of the analysis over data of CMC from The BIPM Key Comparison Database in the field of length are presented in E5.

### Uncertainty tolerance ratios optimization

Now we will briefly touch upon the problem which, although officially is not in frame of the INPL's activity, but is of importance. Being a metrological center, we are often asked by industrial clients and companies about the proper selection of measuring instruments by their uncertainty in various measurement practices and processes. This is the problem of determining rational uncertainty tolerance ratios (UTR).

In applying the proposed conception of informational optimality, the criterion of UTR optimization materially differs from TUR. In this case we shall deal with nominal values ($X_j$) of parameters undergoing measurement, with their specified tolerances ($D_j$) and expanded uncertainties ($U_j$), and not with uncertainties' variances.

Practically, properly established tolerances of parameters are being integral characteristics, determined with taking into account functional properties of an object (a product, a process, a service, etc.), economic and other conditions. That is why the weights, i.e. the indices characterizing the parameters' importance can be determined by the ratios of nominal values of parameters to their tolerances. The magnitude of weight determines the relative contribution of a parameter into the quality of object.

Over the totality of parameters affecting the quality of object, systematically they and their weights according to this approach can be classified and normalized over certain accuracy groups, providing optimal information on the quality intrinsic to each group. Then the accuracy groups can be considered as quality's subsystems; and the procedure of uncertainty tolerance optimization consists in the following:

1) Distribution over accuracy groups ($G$) of all parameters ($X$) being subjected to measurements at all levels of production and/or testing. Each accuracy group is being defined according to the sequence $G = 0, 1, 2, \ldots$ over the following condition for the relative tolerance ($D_j/X_j$) and $\rho_o = 1/2\pi$:

$$G = arg\ [(1/2\pi)^{G+1} \leq (D_j/X_j)_G \leq (1/2\pi)^G], \tag{38}$$

2) At each accuracy-group the calculation of normalized weights ($K_{jG}$) of parameters, identified within the group:

$$K_{jG} = (X_j/D_j)_G \,/ \sum(X_j/D_j)_G \tag{39}$$

3) At each accuracy-group the calculation of optimal uncertainty tolerance ratios ($\mathrm{UTR}_{(jo)G}$) or as suitable over optimality as follows:

$$\mathrm{UTR}_{(jo)G} = (U_j/D_j)_G = (K_{jGmax}/K_{jG})/\,2\pi, \tag{40}$$

where $K_{jGmax}$ = the maximal weight, identified within the accuracy-group and related to the most significant parameter of the group over an impact on quality.

The drawback of formula (40) consists in rising the estimation uncertainty if condition (38) becomes close to equality. The more reliable estimation can be achieved by means of unified uncertainty tolerance ratio $UTR_{(G)}$ in an accuracy group when using for the unification the approximation of the already discussed linear model of weights' diagram ($\Delta K_j = K_j - K_{j+1}$ = constant; $j$ = 1, 2, …, $N$; $K_{max} = 2/(1 + N)$) and the mean of weighs over $\varphi_o = N/(1 + \rho_o)$ informative parameters.

After simple algebraic transformation for the linear diagram, the following expressions are true in this case:

$$UTR_{(G)} = 2[N_G /(1 + N_G)]*[\rho_o/(1 + \rho_o)] = 0.274N_G /(1 + N_G) \quad (41)$$

Graphically $UTR_{(G)}$ as function of $N_G$ is presented in Fig. 7. For the comparison Fig. 6 involves also the most frequently used UTR: 1/3, 1/5, and 1/10, as well as the most pessimistic estimation UTR = $1/2\pi$ in general case (when the parameters of accuracy group are not of the same weight).

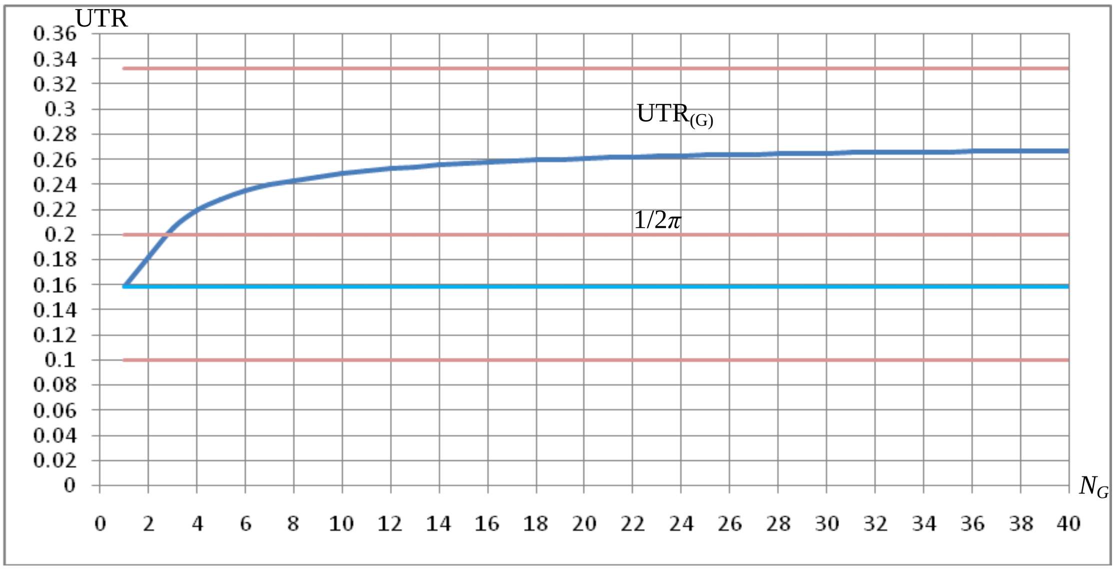


Fig. 7: Graphical illustration of $UTR_{(G)}$ as function of $N_G$

In theoretically exceptional case when all parameters in the group are equal by weights, the calculation results in the same for all parameters $UTR_{(jo)}$ = $UTR_{(o)}$ = $1/2\pi$. Clearly this value is appropriate also when a parameter is considered as being not associated with others in the system (non-classification approach).

## Examples

**E1.** To illustrate the practical application of the proposed method concerning combined and expanded uncertainties we will apply to popular among metrologists examples of uncertainty budgets of EA4/02 [26] and to examples of EURACHEM/CITAC Guide CG 4 [27]. Correspondingly initial data and results of calculations pertaining to both documents are illustrated by two sets of data in certain measurement fields presented in Tables 5 and 6. In particular, one of the results is 40% reduction of the contributing uncertainties ($\Psi$) to be taken into account for determining the combined uncertainty in the first example, and 57% - in the second.

Table 5: Uncertainties in calibrating the 10 kg weight, OIML class M1[26]

| $u_j$ | Source of uncertainty | Standard uncertainty ($u_j$) | Variance ($u_j^2$) | LC$_{oj}$ (%) | $k_j$ |
|---|---|---|---|---|---|
| $u_1$ | Conventional mass of the standard | 22,54 mg | 508.1 | 92 | 1.75 |
| $u_2$ | Drift of the standard since last calibration | 8,95 mg | 80.1 | 50 | 0.67 |
| $u_3$ | Difference between the unknown and standard mass | 14,4 mg | 207.4 | 80.5 | 1.3 |
| $u_4$ | Correction for eccentricity and magnetic effects | 5,77 mg | 33.3 | | |
| $u_5$ | Correction for air buoyancy | 5,77 mg | 33.3 | | |
| $u_c$ | Combined uncertainty (EA-4/02 version) | 29,3 mg | | | |
| $u_{cp}$ | Combined uncertainty (proposed version) | 26.7 mg | | | |
| $U$ | Expanded uncertainty (EA-4/02 version) | 59 mg | | | |
| $U_p$ | Expanded uncertainty//proposed version | 44 mg // 47 mg | | | |
| $u_{\varphi o}^2 = u_1^2/2\pi = 80.9$; $\varphi_o = 3$; $\Psi = [(N- \varphi_o)/N]*100\% = 40\%$; $L_q = 0.026$; | | | | | |
| $k_{in} = [(k_1^2u_1^2 + k_2^2u_2^2 + k_3^2u_3^2)/(u_1^2 + u_1^2 + u_3^2)]^{1/2} = 1.56$; JLC$_{in}$ = 88% | | | | | |

Table 6: Uncertainties in determining the cadmium release from ceramic ware by atomic absorption spectrometry [27]

| $u_j$ | Source of uncertainty | Relative standard uncertainty | Variance ($u_j^2$) | LC$_{oj}$ (%) | $k_j$ |
|---|---|---|---|---|---|
| $u_1$ | Content of cadmium in the extraction solution | 0.069 | 0.00476 | 86 | 1.49 |
| $u_2$ | Volume of the leachate | 0.0054 | 0.0000292 | | |
| $u_3$ | Surface area of the vessel | 0.025 | 0.000625 | | |
| $u_4$ | Influence of the acid concentration | 0.0008 | 0.00000064 | | |
| $u_5$ | Influence of the duration | 0.001 | 0.000006 | | |
| $u_6$ | Influence of temperature | 0.06 | 0.0036 | 82 | 1.34 |
| $u_7$ | Mass of cadmium leached per unit area | 0.09 | 0.0081 | 92 | 1.75 |
| $u_c$ | Combined uncertainty (CG 4 version) | 0.131 | | | |
| $u_{cp}$ | Combined uncertainty (proposed version) | 0.128 | | | |
| $U$ | Expanded uncertainty (CG 4 version) | 0.262 | | | |
| $U_p$ | Expanded uncertainty//proposed version | 0.204 // 0.225 | | | |
| $u_{\varphi o}^2 = u_7^2/2\pi = 0.0013$; $\varphi_o = 3$; $\Psi = [(N- \varphi_o)/N]*100\% = 57\%$; $L_q = 0.012$; | | | | | |
| $k_{in} = [(k_1^2u_1^2 + k_6^2u_6^2 + k_7^2u_7^2)/(u_1^2 + u_6^2 + u_7^2)]^{1/2} = 1.59$; JLC$_{in}$ = 89% | | | | | |

Analogous analysis has been performed over other practical examples found in CG 4 and EA-4/02, and summary results are presented in Table 7.

Table 7: Summary results of analyzing the examples found in EA-4/02 and CG 4

| # | Objects of calibration from EA-4/02 (# 1÷13) and CG 4 (# 14÷19) | $\Psi$ (%) | $LC_{in}$ (%) | $k_{in}$ | $L_q$ (%) |
|---|---|---|---|---|---|
| 1 | Weight of nominal value 10 kg of OIML class Ml | 40 | 88 | 1.56 | 0.026 |
| 2 | Gauge block of nominal length 50 mm | 29 | 90 | 1.64 | 0.018 |
| 3 | Type N thermocouple at 1000$^o$C ($t_x$ $^o$C of the furnace hot junction) | 87 | 92 | 1.75 | 0.029 |
| 4 | Type N thermocouple at 1000$^o$C (emf $V_x$ of the thermocouple) | 86 | 92 | 1.75 | 0.004 |
| 5 | Power sensor at a frequency of 19 GHz | 56 | 88 | 1.56 | 0.003 |
| 6 | Coaxial step attenuator at a setting of 30dB (incremental loss) | 78 | 90 | 1.63 | 0.006 |
| 7 | Hand-held digital multimeter at 100 V (DC) | 67 | 92 | 1.75 | 0.017 |
| 8 | Vernier calliper | 50 | 90 | 1.63 | 0.001 |
| 9 | Temperature block calibraror at a temperature of 180 $^o$C | 75 | 90 | 1.64 | 0.017 |
| 10 | Hausehold water meter (regarding the volume passed the meter) | 87 | 92 | 1.75 | 0.023 |
| 11 | Hausehold water meter (regarding the relative error of indication) | 0 | 88 | 1.54 | 0 |
| 12 | Hausehold water meter (regarding the repeatability of the meter) | 0 | 91 | 1.7 | 0 |
| 13 | Ring gauge with a nominal diameter of 90mm | 57 | 88 | 1.57 | 0.017 |
| 14 | Preparation of a Calibration Standard for Atomic Absorption Spectroscopy (1000mg/l Cd in dilute $HNO_3$) | 33 | 90 | 1.65 | 0.002 |
| 15 | Standardisining a Sodium Hydroxide Solution | 20 | 87 | 1.52 | 0.0002 |
| 16 | An Acid/Base Titration | 43 | 91 | 1.67 | 0.028 |
| 17 | Determination of Organophosphorus Pesticides in Bread | 33 | 90 | 1.65 | 0.01 |
| 18 | Determination of Cadmium Release from Ceramic Ware by Atomic Absorption Spectrometry | 40 | 89 | 1.59 | 0.026 |
| 19 | Determination of the Amount of Lead in Water Using Double Isotope Dilution and Inductively Coupled Plasma Mass Spectrometry | 94 | 92 | 1.75 | 0.021 |

The analysis of above examples over the use of proposed methods demonstrates:

(a) the significant ($\psi \approx 50\%$ in average over the analyzed uncertainty budgets) reduction of uncertainty contributions taken into account when determining the respective combined uncertainty,

(b) rather narrow ranges of intrinsic level of confidence ($LC_{in}$ from 88% to 92%) and respective intrinsic coverage factor ($k_{in}$ from 1.52 to 1.75),

(c) rather small (from 0% to 0.029%) relative quality loss in evaluating measurement uncertainties. This result is very consistent with expected data obtained in previous study related to quality losses.

**E2.** To illustrate the direct (using specified expanded uncertainties at 95% LC) practical application of the proposed method concerning sufficient interval of TURs we have used data of CMC from The BIPM Key Comparison Database [28].There were chosen for the consideration 30 NMIs (having CMC for calibrating gauge blocks in length 0.5mm to 100mm both by the interferometer and the mechanical comparator). Fig.8 and Fig.9 show results of the analysis in test uncertainty ratios and traceability losses respectively.

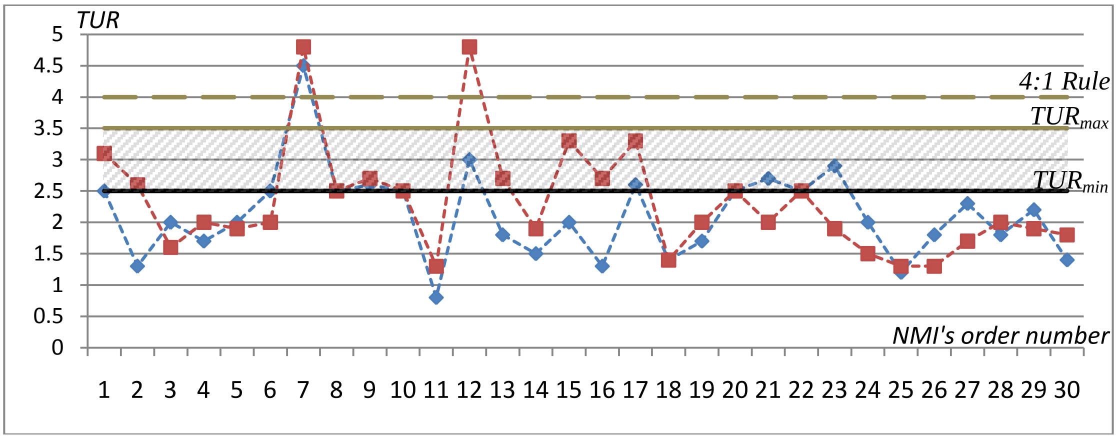


Fig. 8: Test uncertainty ratios of 30 NMIs regarding CMC for gauge blocks 0.5mm (blue marks) and 100mm (red marks). The area between $TUR_{min}$ and $TUR_{max}$ represents TUR estimation sufficiency, whereas TUR< $TUR_{min}$ and TUR> $TUR_{max}$ are characterizing the estimation insufficiency and redundancy respectively.

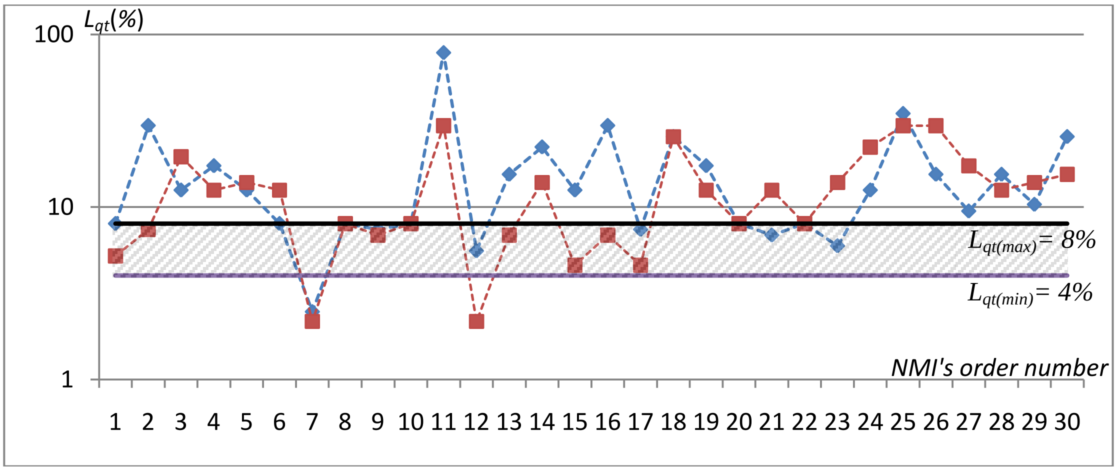


Fig. 9: Traceability quality losses percentages for 30 NMIs regarding CMC for gauge blocks 0.5mm and 100mm. The area between $L_{qt(min)}$ and $L_{qt(max)}$ represents $L_{qt}$ estimation sufficiency, whereas $L_{qt} < L_{qt(min)}$ and $L_{qt} > L_{qt(max)}$ are characterizing the estimation redundancy and insufficiency respectively.

Interestingly, in accordance with the requirements of OIML recommendations [20] for the analyzed gauge blocks TUR = 2.5 that illustrates although the boundary value of TUR but satisfactory to the proposed criterion.

In real practice (as is demonstrated in Fig. 8 and Fig.9) there is the rather non-satisfactory situation for the majority of NMIs with providing the traceability in the considered measurement field. (◊) Clearly, this conclusion as well as data it relies onto requires an additional and deeper analysis using the proposed informational approach and method.

◊ Note *Unfortunately, this situation well correlates with cancelling OIML R 30 [21]*

**E3**. The manifestation of 7±2 phenomenon regarding accuracy classes in calibration systems of length and mass measurements results in the following. The system of gauge blocks calibration consists of five orders of accuracy; numerical analysis of system shows the two TURs = 2.5 = $TUR_{min}$, and two ones equal 2 that is less than $TUR_{min}$. The distribution of nominal weights by OIML accuracy classes, also can exemplify this discussion. OIML R111 [20] irregularly distributes 30 nominal values ($V_n$) of weights (from 1mg to 5000kg) among specified 9 accuracy classes $E_1$, $E_2$, $F_1$, $F_2$, $M_1$, $M_{1-2}$, $M_2$, $M_{2-3}$ and $M_3$ as is shown in Table 8.

Table 8: Distribution of nominal weights over OIML classes of accuracy

| Nominal weights (from – to) | Number of nominals | Classes (from – to) | Number of classes |
|---|---|---|---|
| 1mg – 50mg | 6 | $E_1 – M_1$ | 5 |
| 100mg – 500mg | 3 | $E_1 – M_1$; $M_2$ | 6 |
| 1g – 20kg | 14 | $E_1 – M_1$; $M_2$; $M_3$ | 7 |
| 50kg | 1 | $E_1 – M_3$ | 9 |
| 100kg – 1000kg | 4 | $E_2 – M_3$ | 8 |
| 2000kg – 5000kg | 2 | $F_1 – M_3$ | 7 |

The percentage distribution of nominal weights by OIML classes, presented in Fig. 10, demonstratively shows the conformity with 7±2 phenomenon in this example.

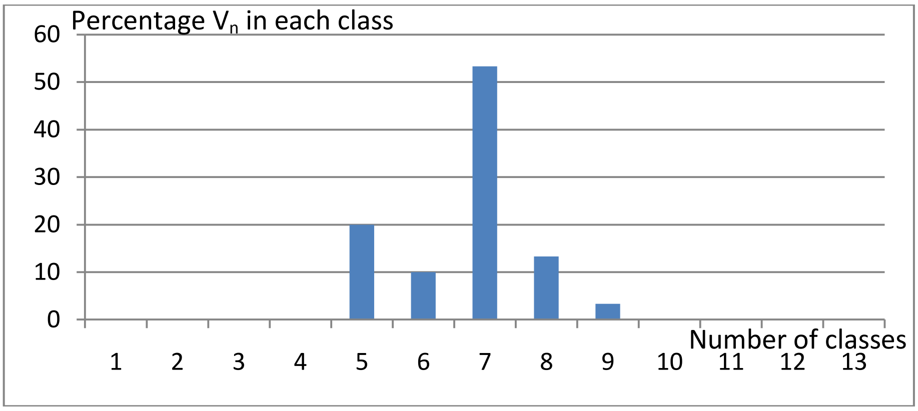


Fig. 10: Percentage distribution of nominal weights by OIML classes

**E4.** To illustrate the new method of determining and correcting measurement traceability we will consider $TUR_r$ determination when calibrating the 1mm gauge block by means of the TESA Comparator, using INPL procedures [29]. The block undergoing calibration is being compared with the INPL standard block, in turn calibrated by means of the INPL TESA Interferometer [30].

In this case traditionally estimated TUR (as the ratio of expanded uncertainties) equals $2u_{c(IC)}/2u_{c(RS)}$ =50nm/30nm = 1.67 that is significantly less than the minimum permissible, i.e. 2.5. After selecting informative uncertainty components by conditions (27) and (28), sets of coverage factors $k_{j(RS)}$ and $k_{j(IC)}$, as well as respective intrinsic coverage factors $k_{in(RS)}$ and $k_{in(IC)}$ have been determined by expressions (32) ÷ (34); all these data are presented in Table 9.

Table 9: Budgets of informative uncertainties and coverage factors specific to the calibration of the 1mm gauge block by TESA Interferometer and Comparator

| TESA Interferometer | | TESA Comparator | |
|---|---|---|---|
| Uncertainty budget (selected) for calibrating 1mm gauge block $u_{j(RS)}$ (nm) | Coverage factors $k_{j(RS)}$ | Uncertainty budget (selected) for calibrating 1mm gauge block $u_{j(IC)}$ (nm) | Coverage factors $k_{j(IC)}$ |
| Fringe fraction: $u_{1(RS)} = 7.5$<br>Surface roughness: $u_{2(RS)} = 6.4$<br>N + $K_o$ with λ: $u_{3(RS)} = 3$<br>Wringing film: $u_{4(RS)} = 5.1$<br>Interferometer optics: $u_{5(RS)} = 5.9$<br>--------------------------------------<br>Combined uncertainty: $u_{c(RS)}$=12.9 | 1.75<br>1.6<br>0.67<br>1.37<br>1.51<br>---------<br>$k_{in(RS)}$ = 1.54 | Calibration of standard block: $u_{1(IC)} = 25$<br>Discrimination and linearity of comparator: $u_{2(IC)}$ =18.5<br>------------------------------------<br>Combined uncertainty: $u_{c(IC)}$= 31.1 | 1.75<br>1.44<br>---------<br>$k_{in(IC)}$ = 1.64 |

Applying Table 9 data and expressions (35), (36), the real test uncertainty ratio is determined as: $TUR_r = [u_{c(IC)}\, k_{in(RS)}] / [u_{c(RS)}\, k_{in(IC)}] = 2.26$. Since the obtained value is still less than the minimally permissible 2.5, in order to achieve the informationally sufficient traceability the CMC for mechanical comparison could be corrected ($u_{c(IC)cor}$) as follows: $u_{c(IC)cor} = 2.5*(u_{c(IC)} / TUR_r) = 34$nm.

**E5.** Applying the same data of CMC uncertainties ($U_i$) from The BIPM Key Comparison Database that have been used in E2, the degree of equivalence of measurement standards maintained by NMIs in the field of length regarding calibration by interferometers and mechanical comparators has been examined. Graphically data and outcomes of the examination by proposed criteria (37) are presented in Fig. 11 and Fig. 12. The obtained results show the following:

(a) in the field of interferometric measurements only 3 (from examined 41) NMIs do not satisfy to the criteria, i.e. are above the level of $U_{rI(1)}\sqrt{2\pi}$ for the calibration of gauge blocks of 0.5mm nominal length (data in red) or/and $U_{rI(2)}\sqrt{2\pi}$ for the calibration of gauge blocks of 100mm nominal length(data in blue);

(b) in the field of mechanical comparisons the situation is significantly worse: 17 (from examined 37) NMIs should be admitted as MRA outliers.

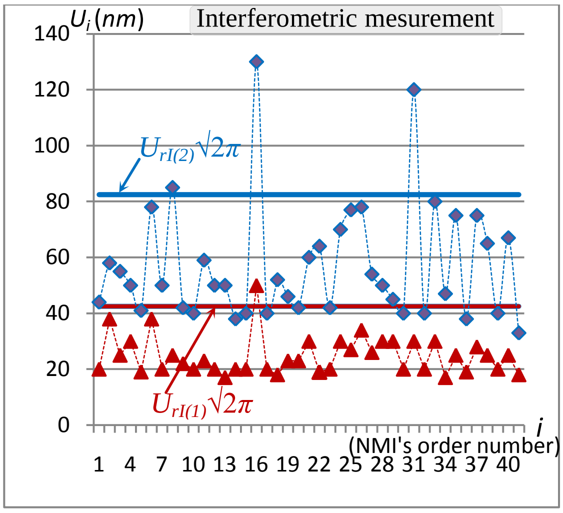


Fig. 11: Graphical data of MRA verification for interferometric measurement

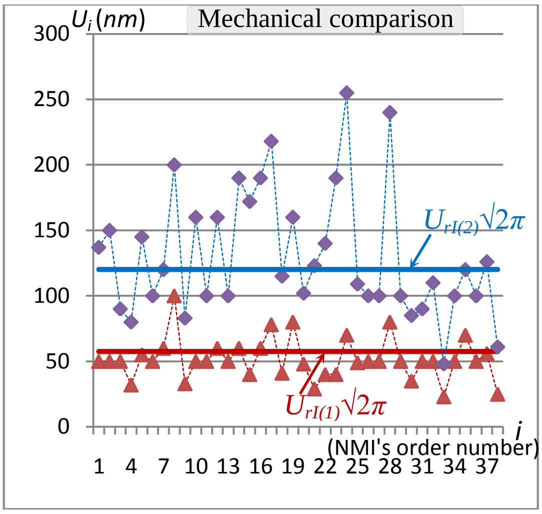


Fig. 12: Graphical data of MRA verification for mechanical comparisons

**E6.** Any system, which quality is characterized by means of different by weights measurement parameters, can be considered as the model of practical exemplifying the proposed method of UTR determination. Regarding the parameters, many objects along with grouping by accuracy possess their specific technological classification which should be taken into account. In manufacturing sphere, for instance, such a classification is connected with isolating relatively independent production stages.

Suppose the quality of some abstract product is being monitored by performing various measurements (in toto 23 parameters) on the stages of receiving inspection (*r*), production process (*p*), and output tests (*o*). Table 8 involves distributed over these stages relative tolerances $(D_j/X_j)_r$, $(D_j/X_j)_p$, and $(D_j/X_j)_o$ of the parameters undergoing measurement, and the calculated in accordance with the proposed procedure and expressions (38), (39), and (40) accuracy groups (*G*) and optimal uncertainty tolerance ratios. In Table 10: $N_o = 14$, $N_1 = 6$, and $N_2 = 2$; and comparative data of TURs calculations by formulas (41) and (40) one may find in Fig. 13.

Table 10: Data and results of $\text{UTR}_{(jo)}$ calculation in manufacturing the abstract product

| *j* | $(D_j/X_j)_r$ | *G* | $\text{UTR}_{(jo)}$ | $(D_j/X_j)_p$ | *G* | $\text{UTR}_{(jo)}$ | $(D_j/X_j)_o$ | *G* | $\text{UTR}_{(jo)}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.35 | 0 | 0.33 | 0.3 | 0 | 0.29 | 0.2 | 0 | 0.19 |
| 2 | 0.19 | 0 | 0.18 | 0.3 | 0 | 0.29 | 0.25 | 0 | 0.24 |
| 3 | 0.5 | 0 | 0.48 | 0.25 | 0 | 0.24 | 0.1 | 1 | 0.31 |
| 4 | 0.01 | 2 | 0.16 | 0.1 | 1 | 0.31 | 0.11 | 1 | 0.34 |
| 5 | 0.17 | 0 | 0.16 | 0.4 | 0 | 0.38 | 0.3 | 0 | 0.29 |
| 6 | 0.12 | 1 | 0.36 | 0.05 | 1 | 0.16 | | | |
| 7 | | | | 0.3 | 0 | 0.29 | | | |
| 8 | | | | 0.3 | 0 | 0.29 | | | |
| 9 | | | | 0.2 | 0 | 0.19 | | | |
| 10 | | | | 0.09 | 1 | 0.29 | | | |
| 11 | | | | 0.1 | 1 | 0.31 | | | |
| 12 | | | | 0.015 | 2 | 0.24 | | | |

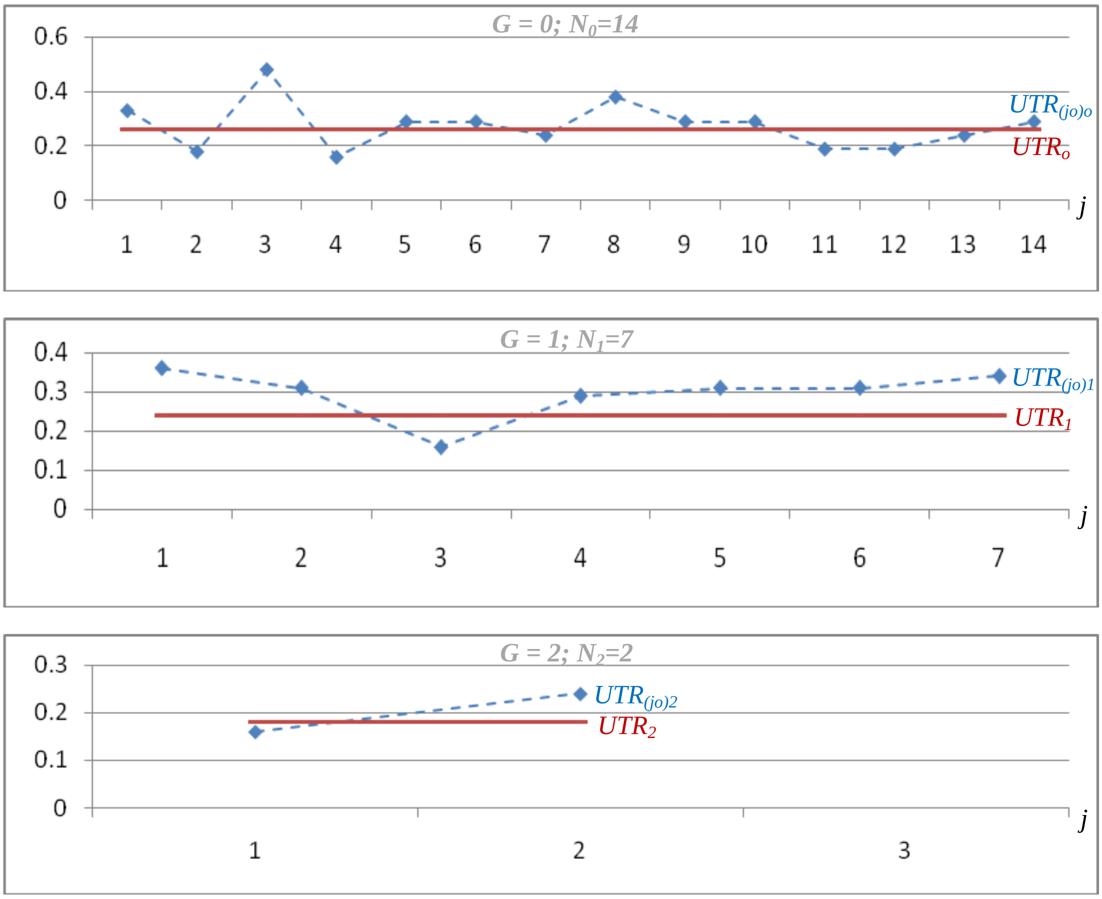


Fig. 13: Data of UTR calculation by formulas (40) – markers, and (37) – solid line, distributed over accuracy groups $G$ = 0, 1, and 2 in the example of abstract product

## Conclusions

1. We have discovered - and this discovery is the foundation stone of applied metrology science - that by employing PNRs as numerical classification criteria of informational optimality, of harmony and balance, unequivocal requirements to basic characteristics in evaluating measurement uncertainty, i.e. informative uncertainty components, levels of confidence, and measurement traceability (test uncertainty ratios) can be unambiguously determined.

2. The revealed comparative closeness of the optimal quantitative criteria and the criteria that have been formed and commonly used as the result of persistent metrology practice demonstrates that the last ones reflect the natural evolutionary trend to optimization.

3. The identification of informative uncertainty components in analyzing an uncertainty budget and the establishment of informatively optimal level of confidence for expanded uncertainty are becoming greatly concretized when using elements of qualimetry and information theory. The major criterion of doing this is the informatively optimal classification ratio of the variance of an uncertainty contribution and the variance of maximal uncertainty contribution. This ratio should be not less than $1/2\pi$.

4. The informational treatment of dimensional factors of confidence has allowed optimazing levels of confidence for the evaluation of measurement uncertainty. It follows from informational classification that along with 68% confidence, related to one standard deviation of normal distribution, there are critical levels of confidence for the weightiest component of a classification group: approximately 92%, 95%, and 96%. These LCs are strictly associated with conceptions of information optimality, of harmony, and of balance respectively and are characterized by the following classification qualities regarding informational redundancy:

92% level of confidence is related to the boundary of redundancy absence;

95% level of confidence is related to the optimal redundancy;

96% level of confidence is related to the maximum permissible redundancy.

5. The analysis of coverage factors for normal distribution by means of the proposed method of LCs optimization has revealed the conformity of the factors with theoretically stated informatively permissible ranges and redundancy, and the dimensional perfection of critical coverage factors' ratios in terms of optimality, harmony and balance expressed by perfect numerical ratios.

6. The possibility of using either intrinsic or common LC in specifying an expanded uncertainty is demonstrated. Informational analysis has resulted in the conclusion that

(a) the commonly used estimating rates: $u_j/u_{max}$ = 1/3 and LC = 95% are located within respective informatively permissible ranges of accuracy classification, but are characterizing certain information redundancy;

(b) informatively sufficient interval of TUR lies between 2.5 and 3.5, and in this instance the practicing recommendation called '4:1 Rule' possesses information redundancy. This outcome makes doubtful the popular idea that “a 4:1 TUR is the point to which most high-quality calibration labs strive” [31].

7. The model of informatively perfect measurement traceability chain has been proposed and substantiated.

8. Effectiveness of the new approach of identifying informative uncertainty components has been determined with the aid of subsidiary estimation criterion, called relative quality losses, and illustrated by a series of practical examples. The criterion itself has been subjected to the study regarding boundary models of weights' diagrams that has been resulted in detecting theoretical ranges of relative quality losses and their approximate expectations in real practice.

9. The significant judgment ought to be done over CMC: the correct determination and specification of calibration and measurement capability (as the Lab's declaration) demands its conformity to the established quantitative criterion of measurement traceability. Evidently this rule should be incorporated into ISO/IEC 17025 and other international and regional standards regulating the activity of calibration and testing laboratories, and their accreditation.

10. The importance of analyzing existing test uncertainty ratios and their improvement to provide the informatively sufficient measurement traceability was demonstrated on the example of length measurement. The detailed practical procedure of establishing a realistic traceability and its well-founded correction is proposed. It is demonstrated also that the procedure allows determining well-founded CMC.

11. The quantitative criterion for MRA verification has been proposed to establish the degree of equivalence of national measurement standards maintained by NMIs.

12. Simple estimation methods based on the proposed approach have been discussed as useful for calibration and testing laboratories, as well as for widest range of organizations dealing with applied metrology. The difference between test uncertainty ratios and uncertainty tolerance ratios has been clarified, and the method and procedure for UTR optimization have been proposed. One of notable outcomes is that the optimal $UTR_o = 1/2\pi$ of the weightiest parameters or of a parameter considered beyond the system is within the most commonly used [32] range 1:5 and 1:10.

13. Two additional results are obtained. Firstly this is the substantiation of Miller's 7±2 phenomenon, and the demonstration of its significance in establishing optimal and rational accuracy classifications, and secondly the proved interrelations of PNRs. As for the second outcome, the obvious connection of $f_o$ and $\lambda$ with $\rho_o$, as well as the perception of mathematical harmony and balance in terms of dimensional ratios, with good reason allows considering $f_o$ and $\lambda$ as information constants alike $\rho_o$. On the whole the constants shall help in analyzing various ratios in nature and human activity, including metrology, over a degree of conformity to dimensional perfection.

## Follow-up discussions and recommendations

1. Clearly the proposed method of selecting and combining measurement uncertainty contributions is rather effective and does not need additional substantiation. However, the excluding of redundant uncertainty contributions from the calculation of combined uncertainty not always means their ignoring in certain technology of measurement. This is true because in some cases certain measurement conditions should be controlled even when acceptable to be in rather wide ranges. Then, if needed, measuring instruments servicing such conditions can rank among measurement indicators and need much more simple maintenance.

1.1. The proposed and substantiated formula of determining necessary and sufficient confidence level for each uncertainty contribution and the conducted theoretical analysis make possible asserting that the commonly used unitary mode of expanding measurement uncertainty on the level of 95% confidence in fact provides informational optimality in terms of optimal redundancy for the weightiest uncertainty component only, and not for the system of selected components as a whole. This means the expanded uncertainty possesses unsubstantiated pessimistic overestimation. The usage of 92% level of confidence is more realistic, but is characterized by the same principle drawback, although in lesser extent.

1.2. The passing from the unitary mode to the proposed intrinsic expansion of uncertainty leads to the optimization matching the main criterion; and this is accompanied by the complete changing of philosophy of determining the expanded uncertainty and provides the users with individual information on a measurement method. Clearly such decision requires wide discussion among metrologists and possibly of additional (preferably economic) substantiations, and could be taken as the result of consensus. In the Author's opinion, one of significant innovations is the proved necessity for determining CMC by taking into account the informatively permissible TUR, in turn determined over uncertainties, levels of confidence and coverage factors intrinsic for measurement methods used and needed to be additionally specified in calibration certificates.

1.3. Further development of the discussed approach is reasonable to be carried out in the direction of taking into account correlated/interrelated sources of measurement uncertainty. If the use of variance analysis (ANOVA) or other similar technique is possible, then one of ways of calculating weights for correlated uncertainty components may be carried out as proposed in [33, 34].

2. As has been fairly noted in EAL-G12 [35], "if binding requirements for the accuracy of measuring and test equipment have been stipulated, failure to meet these requirements means the absence of a warranted quality with considerable consequent liability". The proposed new approach and criteria in establishing TUR as a traceability characteristic, as well as the results of carried out examination on the real example may eliminate often in practice negative situations with traceability, and can be an impulse for further analyses and improvement in various fields of measurement. In the author's opinion, some traceability problems at highest accuracy level might be solved using the intrinsic LC of reference standard on the one hand and 95% or, if needed, 96% LC for the object undergoing calibration on the other hand.

3. It is not difficult to realize that beyond calibration activity in wide practice of measurement the proposed ideas and developed on their basis the methods of optimizing measurement control can serve as an effective instrument of decreasing technological expenses. It may be not out of place to note: for an advanced industrial economy it is estimated that between 3% and 6% of gross domestic product (GDP) is accounted for by measurement and measurement-related operations [36]. While the optimization of TURs might be called as *ensuring quality in controlling measurement accuracy* (EQCA) in calibration and testing laboratories relevant to requirements of ISO 17025, for all other objects, e.g. a product, a process, a service, etc. EQCA shall base on the optimization of uncertainty tolerance ratios (UTR). The proposed procedure of doing this one ought to be considered as just one of possibilities, so the problem needs further development.

4. The analysis of obtained largely theoretical results leads naturally to interesting question: does an emphasis on theory or theoretical awareness mean the expectation of more agreement or disagreement among metrologists? In the author's opinion the proposed ideas can be a basis for wide discussion among specialists involved in metrology and fields dealing with measurement for improving the ways of uncertainty evaluation. Significant updating of international documents specifying measurement uncertainty evaluation, as well as acting in calibration and testing laboratories standard operational procedures might be initiated as the result of such a discussion.

5. The further creation of common approach, based on informational classification can be predicted, which is aimed at solving significant accuracy problems in applied metrology that along with discussed in frame of this work include tolerance uncertainty ratios, accuracy classes and traceability chains, etc.

6. It should be noted the practical value of the proposed criteria have been already demonstrated before for interlaboratory comparisons' activity and in developing of and participating in proficiency testing programs [12]. This touches upon performance indicators Z scores and E numbers, which are most often used for the treatment of

proficiency test data. In the Author's opinion a further unification of this problem with those discussed in the Report based on the same approach might form the complete scientifically substantiated system for the activity of calibration and testing laboratories over measurement accuracy problematic - on the one hand, and in the activity connected with their accrediting - on the other hand.

7. Both beyond and in frame of metrology it is reasonable to pay attention at least onto the following PNR peculiarities that might be useful in further research:

7.1. The connection between balance and harmony, which has been revealed with the aid of Fibonacci numbers ratios, has made possible to assert that in the nature the harmony is the result of evolution. On the way of evolution the compulsory early or initial stage is the balance. Surprisingly in this regard is that, taking into account the proved 8% permissible deviation of a PNR, Fibonacci succession comprises ratios of balance and harmony only which, in turn, are being characterized by deep informatively optimal connection via $\rho_o$.

7.2. Clearly, PNRs represent the effective criterion for revealing to what extent evolutionary developing quantitative qualities are reaching certain informational perfection, no matter to which nature they belong. This is specifically of high importance in various human practice of estimating of classification character. A detection of significant deflections from certain PNR might indicates onto imperfection of specified estimation rates and thus to the necessity of their improvement. Thus, PNRs may be considered as the informatively sound universal numerical marks of achieving critical stages of evolution's perfection, irrespective of an object undergoing estimation.

7.3. Some classifications of hierarchical type, for instance classes of accuracy in industry and metrology, show to us the definiteness of introduction of both a quantitative measure and cyclicity or periodicity. Necessary and sufficient number (classification integer) of components, groups, categories, or classes is one of problems that hypothetically could be solved or be substantiated, using the proposed criteria in order to provide the optimal or permissible periodicity.

8. While the informational optimization is mandatory, any economic and 'risk analysis' criteria (ERAC) for the suitable secondary optimization of uncertainty (and CMC, in particular) are acceptable within the accuracy classification only. The classification approach demands ERAC should be under quantitative limitation.

9. The introduction of proposed advanced approach as a whole represents an utterly systemic problem that demands much time and efforts, including the needed improvements mentioned above, and the development and adaptation of respective documented methods. At the same time, this activity can and, in the Author's opinion, should be started yet for correcting the already established CMCs. This will serve as an important step in eliminating the defects in providing the traceability and uniformity in all measurement fields. In frame of INPL’s activity the management system ought to be substantially improved by means of additional developing and introduction in the laboratory's practice of respective quality control procedures (QCP). On this basis the updating of acting standard operational procedures (SOP) is also to be carried out.

## Appendix 1: Sufficient number of components in a classification group

The substantiation of the criterion of determining $\varphi_o$ and $\rho_o$ for poly-component system ($N$ >2) traces to the following. Any system of components, which is being analyzed by their weights (significances for certain usage) possesses redundancy (except when all components are of the same weight). The dividing of components onto necessary and sufficient ones on the one hand and redundant on the other hand is the typical act of classification. The criterion of such classifying is the most important problem that can be easily solved using the entropy's approach in terms of theory of information.

In a system of $N$ components the normalized weights are considered as analogs of probabilities that enables using the entropy's approach; and the system's entropy is $H_s = -\sum_{i=1}^{N} K_j \ln K_j$. For the selected by weights optimal (necessary and sufficient) number of components $\varphi_o$ all these components belong to the same classification group, i.e. they are equilibrated by the appertaining: each the component possesses equal *group weight* = $1/\varphi_o$ (otherwise the group could contain inadmissible classification redundancy, i.e. non-optimal selection). Thus, the *classification entrop*y of selected group is $H_c = \ln \varphi_o$.

Theoretically the criterion to be formulated is based on the equivalence of the entropy of initial system and the classification entropy, i.e. $H_s = H_c$ that results in the expression:

$$\varphi_o = \exp\left(-\sum_{i=1}^{N} K_j \ln K_j\right) \quad (42)$$

Then the optimal classification coefficient to be used as the criterion in terms of PIC is being expressed as follows:

$$\rho_o = 1 - (\varphi_o/N)_L = 0.16 \approx 1/2\pi,$$

where: $(\varphi_o/N)_L = 0.5[min(\varphi_o/N)_L + max(\varphi_o/N)_L] = 0.840$,

$$min(\varphi_o/N)_L = \lim_{N\to\infty} [(1/N) \exp(-\sum_{i=1}^{N} K_j \ln K_j)] = 0.824;$$

$$max(\varphi_o/N)_L = min(\varphi_o/N)_L + \lim_{N\to\infty} \sum_{i=1}^{N} K_j - min(\varphi_o/N)_L]^{j+1} = 0.856;$$

$K_{Lj} = \frac{2(N+1-j)}{N(N+1)}$ is the weight in a linear diagram of weights, i.e. when

$$\Delta K_j = K_j - K_{j+1} = \text{constant}.$$

Optimal classification ratio is limited between (1 - $max(\varphi_o/N)_L$) and (1 - $min(\varphi_o/N)_L$), i.e. possesses the range $\rho_o \pm 0.1\ \rho_o$ (that is $0.5/\pi \pm 0.05/\pi$ or $0.159 \pm 0.0159$). In practice the usage of more universal criterion is convenient, according to which $\varphi_o$ is determined as the number of components with weights $K_j \geq K_\varphi$, where $K_\varphi = \rho_o K_1$. This became possible after proving the linear diagram of weights as the proper equivalent of using the criterion for the infinite variety of weights diagrams [13].

## Appendix 2: Permissible PNR deviations

If $\pm 0.5\delta$ is the estimation error in determining a classification constant $C_{inf}$, one may prove that this allows determining whether any practical manifestations conform to the constant within the optimal range $\delta_o$ that is defined as $1/2\pi$ of the basic constant, i.e. $\delta_o/C_{inf} = 1/2\pi$. For this purpose we by analogy with the proposed above way of analysis, expressions (3) ÷ (6) shall proceed to the above method and consider the following equations system:

$$\begin{cases} K_1 = (C_{inf} - 0.5\delta) / (C_{inf} + 0.5\delta); & (43) \\ K_2 = \delta / (C_{inf} + 0.5\delta); & (44) \\ \varphi_o = exp\,(-K_1 \ln K_1 - K_2 \ln K_2) = 1.5 & (45) \end{cases}$$

The solution of these equations system regarding the sought permissible range of estimation error, i.e. $\delta_o = arg\,[\varphi_o(\delta) = 1.5]$, results in the following: $\delta_o = 0.026$ for $C_{inf} = \rho_o$; $\delta_o = 0.098$ for $C_{inf} = f_o$; $\delta_o = 0.080$ for $C_{inf} = \lambda$, and correspondingly with high estimation accuracy the proportion $\delta_o/C_{inf} = 1/2\pi$ is true. Thus, classification constants with their permissible deviations (approximately ±8% of any the constant) are determined as follows:

$\rho_o \pm \rho_o /4\pi = 0.159 \pm 0.013$,
$f_o \pm f_o /4\pi = 0.618 \pm 0.049$,
$\lambda \pm \lambda /4\pi = 0.5 \pm 0.04$

These values can be taken as tolerated ones by the criterion of acceptability in determining the conformity to the requirements of the optimality, harmony and balance. A deflection from the tolerances demonstrates how much an analyzed object does not meet respective PNR, and is the stimulus for more deep investigation of the object or existing estimation criteria and method used.

## Appendix 3: Sufficiency, interrelations and separability of PNRs' system

The dimensional perfection as the system consisting of three independent components with respective weights ($W$) can be represented as a complex quality. Then the condition of non-redundancy, i.e. of including all $n$ components into informative ones can be expressed as $(3 - \varphi) \leq 0.5$, where: $\varphi = - exp(W_1 \ln W_1 + W_2 \ln W_2 + W_3 \ln W_3)$, and $W_j = C_j /( C_1 + C_2 + C_3)$.

The equality in the condition matches the most uncertain classification situation (50% confidence) about allowing or ignoring the system's component with lesser weight. Thus, satisfying the condition, a system under consideration does not contain inadmissible redundancy, in other words the system possesses informational sufficiency.

There is the duality in analyzing the dimensional perfection in such a way that consists in the necessity in considering four separate systems ($s_1 \div s_4$); each one contain $n$ = 3 independent components $C_1$, $C_2$ and $C_3$ so that $C_1$ equals either $\rho_o$ or $(1 - \rho_o)$, $C_2$ equals either $f_o$ or $(1 - f_o)$, and $C_3 = \lambda = (1 - \lambda)$. In so doing, results of calculation, embracing all combinations due to the duality, are presented in Table 11.

Table 11: Weights and sufficiency of dimensional perfection system

| $s_i$ | $C_1 = \rho_o$ | $C_1 = 1-\rho_o$ | $C_2 = f_o$ | $C_2 = 1-f_o$ | $C_3 = \lambda$ | $W_1$ | $W_2$ | $W_3$ | $\varphi$ | $n - \varphi$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.159 | | 0.618 | | 0.5 | 0.125 | 0.484 | 0.391 | 2.61 | 0.39 |
| 2 | 0.159 | | | 0.382 | 0.5 | 0.153 | 0.367 | 0.48 | 2.74 | 0.26 |
| 3 | | 0.841 | 0.618 | | 0.5 | 0.429 | 0.316 | 0.255 | 2.93 | 0.07 |
| 4 | | 0.841 | | 0.382 | 0.5 | 0.488 | 0.222 | 0.29 | 2.84 | 0.16 |

In so far as the above condition is satisfied for all the systems in question, i.e. there are no redundant components, the optimality, harmony and balance represent the complete set of dimensional ratios, i.e. wholly possess informational sufficiency.

The universality of informational optimality is of significance not only in determining permissible ranges for PNRs, but also when considering the hypothesis of their interrelations that we will discuss briefly both for one-dimensional and poly-dimensional models. One-dimensional model can be analyzed using Fig. 14.


Fig. 14: One-dimensional model for analyzing PNRs

Applying to ratios of parts of the straight line $0$–$a_3$, one can easily prove the explicit quantitative interrelation between all three conceptions of dimensional perfection.
If the mean ratio $(a_3 - a_2)/0.5(a_3 + a_2)$ equals $\rho_o$ and the following equations are true

$$(a_3 - a_2)/a_2 = \rho_o (1 + 1/4\pi) = 0.172; \quad (46)$$
$$(a_3 - a_2)/a_3 = \rho_o (1 - 1/4\pi) = 0.146, \quad (47)$$

then such a dimension $a_1$ (approximately $a_1 = a_3 /3$) exists that satisfies to following conditions:

$$(a_2 - a_1)/a_2 = f_o\ (1 \pm 1/4\pi) \approx 0.618 \pm 0.049; \quad (48)$$
$$(a_2 - a_1)/a_3 = \lambda\ (1 \pm 1/4\pi) \approx 0.5 \pm 0.04 \quad (49)$$

Clearly these expressions match permissible PNR deviations. Thus one can assert about the dimensional connection of relative indexes of harmony and balance on the one hand and boundary values of relative index of informational optimality on the other hand. Recently published results of statistically analyzing critical temperatures of the Elements [37] are one of examples of the manifestation of PNRs and of their interrelations in nature. The revealed in the present study PNRs interrelations regarding levels of confidence is another example illustrating one-dimensional model.

As for poly-dimensional systems, any $n$-dimensional model is being mathematically described as a power of exponent ($n$). In our habitual world the numerical ratios geometrically are associating with one-, two- or three-dimensional model, i.e. with a line ($n$ = 1) or a section ($n$ = 2) or a volume ($n$ = 3) respectively (Fig. 15). We shall deal with simplest unitary models: a straight line by length $x = 1$; a square by area $x^2 =$ 1; a cube by volume $x^3 = 1$. The optimal ratio for each model may be presented as $\rho_o = y^n/x^n = y^n$, where $0 \le y \le 1$.Therefore, through classification duality one may consider the functions: $y = \rho_o^{1/n}$ or else $y' = 1 - \rho_o^{1/n}$. Corresponding results of calculations are presented in Table 12.

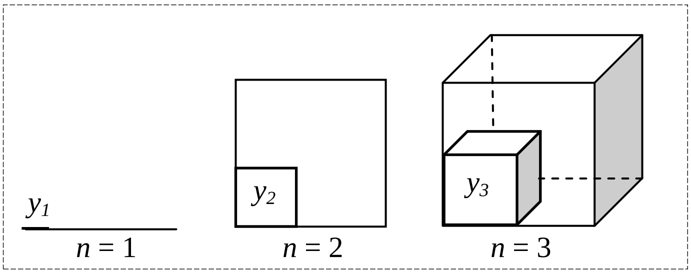

Fig. 15: Unitary dimensional models

Table 12: $y$ and $y'$ values

| $n$ | $y = \rho_o^{1/n}$ | $y' = 1 - \rho_o^{1/n}$ |
|---|---|---|
| 1 | 0.159 | 0.841 |
| 2 | 0.399 | 0.601 |
| 3 | 0.54 | 0.46 |

The following conclusions can be drawn when using such a geometric simulation:

- The obvious connection exists between informational optimality on the one hand and the harmony and balance on the other hand.
- Informational optimality for two-dimensional model ($n$ = 2) results in $y$ = 0.399 and $y'$ = 0.601 that is within the range $f_o \pm f_o /4\pi$ and illustrates mathematical harmony (golden section) adequate to flat models.
- Informational optimality for three-dimensional model ($n$ = 3) results in $y$ = 0.54 and $y'$ = 0.46 that is within the range $\lambda \pm \lambda /4\pi$ and illustrates the balance inherent for volumetric models.
- Thus, depending on the type of normalized geometric model, informational optimality represents the optimality itself for one-dimensional model, and is adequate to the harmony for two-dimensional, and the balance for three-dimensional model.

*PNRs dimensional separability*

The system of PNRs itself can be considered as a one-dimensional classification and, therefore, should match the so-called first rule of classification: to avoid *cross-classification* [38]. One can prove the system possesses this important quality that, taking into account obtained PNRs' tolerances, can be called *dimensional separability*.

An expected characteristic feature of basic and indirect information constants is that in the ranges of their permissible values ($C_{inf} \pm \Delta C_{inf}$), accompanied by permissible estimation errors ($\pm \rho_o \Delta C_{inf}$), they do not have zones of mutual overlap on the axis of the values. If such feature exists, this excludes estimation uncertainty in determining what dimensional perfection the analyzed ratio belongs to. In the set of basic and indirect information constants the feature bears upon adjacent components $C_{inf} = C_{a1}$ and $C_{inf} = C_{a2}$, where $C_{ia2} > C_{a1}$. In so doing, the following condition regarding the difference ($D$), connected with adjacent components, indicates onto the dimensional separability:

$$D = (C_{a2} - \Delta C_{a2} - \rho_o \Delta C_{a2}) - (C_{a1} + \Delta C_{a1} + \rho_o \Delta C_{a1}) > 0 \qquad (50)$$

Data and calculation results proving the reliable dimensional separability are set out in Table 13.

Table 13: Data and results of calculating the differences of adjacent components

| $C_{a1}$ | $C_{a2}$ | $\Delta C_{a1}$ | $\Delta C_{a2}$ | $D$ |
|---|---|---|---|---|
| $\rho_o = 0.159$ | $\lambda = 0.5$ | 0.013 | 0.04 | 0.280 |
| $\rho_o = 0.159$ | $(1 - f_o) = 0.382$ | 0.013 | 0.049 | 0.151 |
| $(1 - f_o) = 0.382$ | $\lambda = 0.5$ | 0.049 | 0.04 | 0.015 |
| $\lambda = 0.5$ | $f_o = 0.618$ | 0.04 | 0.049 | 0.015 |
| $\lambda = 0.5$ | $(1 - \rho_o) = 0.841$ | 0.04 | 0.013 | 0.280 |
| $f_o = 0.618$ | $(1 - \rho_o) = 0.841$ | 0.049 | 0.013 | 0.151 |

One can note the materially small difference ($D = 0.015$) for adjacent components of harmony and balance. Apparently this corresponds with a perception closeness of these conceptions.

## Appendix 4: Harmony and balance as informational conceptions

In order to verify the hypothesis that the harmony and balance rank among the informational conceptions in terms of Shannon's theory one can proceed to entropies regarding $C_{inf}$, which can be determined as $H(C_{inf}) = -K_1(C_{inf}) \ln [K_1(C_{inf})] - K_2(C_{inf}) \ln [K_2(C_{inf})]$, where $K_1(C_{inf}) = C_{inf} /(1 + C_{inf})$, and $K_2(C_{inf}) = 1/(1 + C_{inf})$. Then the quantitative criteria of the verification is whether or not 1) all quantities of information $I(C_{inf}) = H_{max} - H(C_{inf}) = \ln 2 - H(C_{inf})$ belong to the same informational class, and 2) the ratios of entropy $H(\rho_o)$ to $H(f_o)$ and to $H(\lambda)$ are in mathematically harmonious relation.

Graphical illustration of respective calculations' results that prove the satisfaction to the first criterion is presented in Fig. 16, where in coordinates of a ratio ($r$) as a decimal fraction and a quantity of information $I(r)$ one can find locations of $I(\rho_o)$, $I(\lambda)$ and $I(f_o)$ in relation to the following boundary levels: $I(r_b)_S = I(\rho_o)/2\pi$ – the maximum permissible loss of information, and $I(r_b)_R = I(\rho_o)/4\pi$ – the maximum permissible redundancy of information about the quantity $I(\rho_o)$.

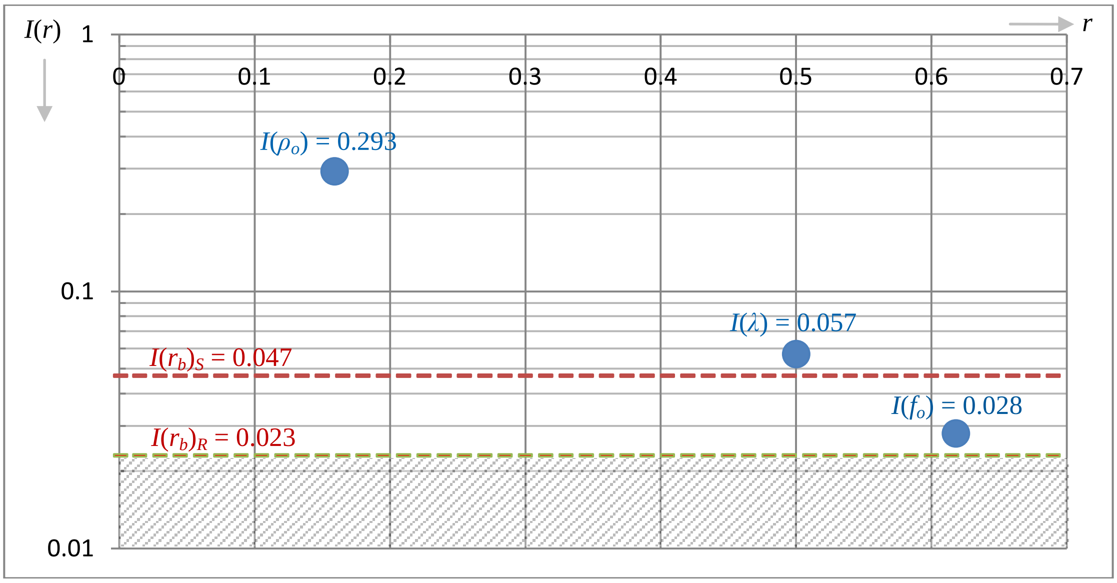


Fig.16: Graphical illustration of informational character of mathematical harmony and balance by their belonging to the same class with informational optimality. The shaded part illustrates the non-admissible redundancy. Any $I(r)$ within this area is not being in frame of the considered class of information quantities

Here are results of calculation as regards the second criterion: $H(\rho_o)/H(f_o) = 0.600$, and $H(\rho_o)/H(\lambda) = 0.627$; they demonstrate the full satisfaction of the condition that the entropy ratios are within the permissible range of harmony = $f_o$ (1 ± 1/4$\pi$). Besides, the ratio $I(f_o)/I(\lambda) = 0.49$ demonstrates a balance of these two quantities within the permissible range $\lambda$ (1 ± 1/4$\pi$).

Thus, along with $\rho_o$ the mathematical harmony $f_o$ and the balance $\lambda$ ipso facto of their deep interrelations over informational properties can be considered as specific informational constants, useful for various quantitative quality estimates.

**Appendix 5: PNR manifestations in critical temperatures of the Elements** [37]

First, before analyzing the relations of critical temperatures of the Elements with the dimensional approach, the system of best (in terms of informational perfection) parameters of normalized statistical distribution serving as the analytical model is suggested below.

*Analytical model*

In the field of statistics it is easy to demonstrate the existence of approximate (2.9% estimation error) harmonious relation: $2\sigma(x)/\mu(x) = f_o$ between double standard deviation (*standard interval* or *standard tolerance*) $2\sigma(x)$ and expectation $\mu(x)$ in the distribution of normalized (sum =1) quantities ($x$) when it is characterized by $\sigma(x) = \rho_o$ and $\mu(x) = \lambda$. By the way, the ratio $2\sigma(x)/\mu(x)$ being double coefficient of variability may be called the *form index* ($\gamma$) of a distribution. Such optimum, harmonious and balanced system is being characterized by the triple perfection quality and in this sense represents informatively unique statistical distribution. On this basis a distribution can be analyzed whether it possesses such triple quality or not.
Thus, if the mean $\mu_e$ and the experimental standard deviation $\sigma_e$ of some real normalized distribution lie within the ranges: $0.46 \leq \mu_e \leq 0.54$ and $0.146 \leq \sigma_e \leq 0.172$ respectively, then likely the harmonious relation between $2\sigma_e$ and $\mu_e$, i.e. their averaged ratio $\gamma_e = 2\sigma_e / \mu_e$ within the range $0.569 \leq \gamma_e \leq 0.667$ can be expected. Now it is possible to proceed to critical temperatures of the Elements as such.

*Statistical system of critical temperatures of the Elements*

The system of melting ($t_m$) and boiling ($t_b$) temperatures of the Elements (Fig. 17) together with the absolute temperature ($t_1$) serves as an example of such statistical regularities in the nature where (and it will be proven below), beyond question, the triple perfection quality exists.

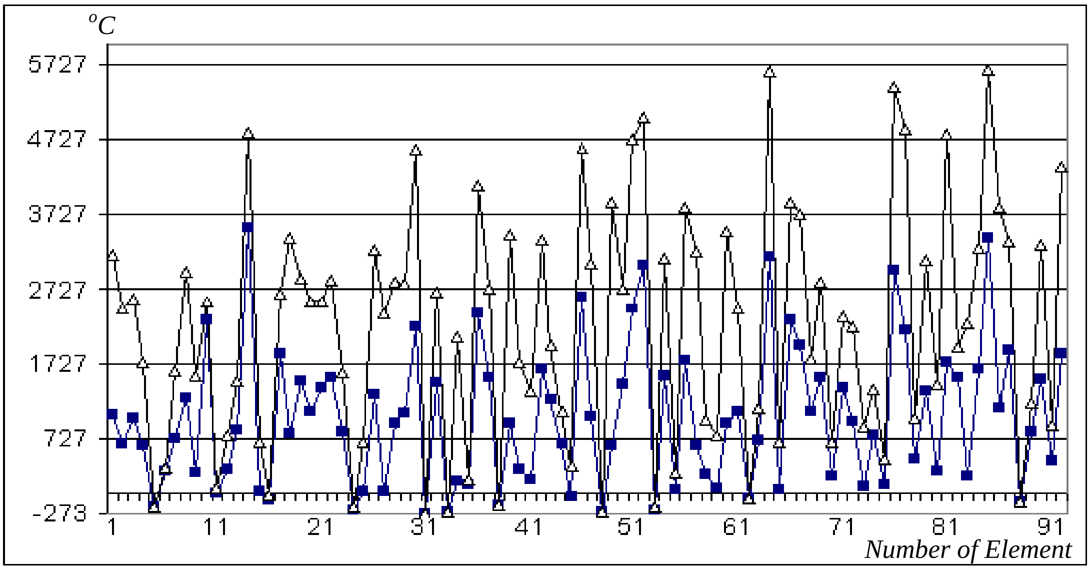


Fig. 17: Distribution of melting (squares) and boiling (triangles) temperatures of chemical elements in their alphabetic order (from Actinium to Zirconium)

The ratios $\alpha_i = (t_b - t_m)/(t_b - t_1)$ and $\beta_i = (t_m - t_1)/((t_b - t_1)$ were calculated as relative dimensional arguments in determining normalized experimental statistical parameters. The critical temperatures of initial group of 93 Elements have been treated to find out and exclude informatively insignificant ones. Possessing quantitative symmetry, a mean ($\mu_e$) according to classification approach predetermines informatively significant components of a system in the limits from $\mu_e * \rho_{oc}$ to $[\mu_e + \mu_e (1 - \rho_{oc})]$. Therefore, the following conditions are true for excluding non-informative Elements:

$\mu_e(\alpha)/2\pi \geq \alpha_i \geq \mu_e(\alpha)*(2 - 1/2\pi)$,

$\mu_e(\beta)/2\pi \geq \beta_i \geq \mu_e(\beta)*(2 - 1/2\pi)$

The calculations performed using routine statistical expressions for determining the means and experimental standard deviations have led to conclusion that, excepting Helium, inert gases (Group Zero of Periodic Table of the Elements), as well as Gallium (Group Thirteen) and Astatine (Group Seventeen) do not meet these requirements and, thus, they ought to be excluded from the consideration. Results of calculation for the rest 86 Elements are as follows:

$\mu_e (\alpha) = 0.486$; $\mu_e (\beta) = 0.514$; $\sigma_e (\alpha) = 0.159$; $\sigma_e (\beta) = 0.159$;

$\gamma_e (\alpha) = 2\sigma_e (\alpha)/\mu_e (\alpha) = 0.654$; $\gamma_e (\beta) = 2\sigma_e (\beta)/\mu_e (\beta) = 0.619$

The closeness of normalized statistical parameters of the system of melting and boiling points of the Elements to information constants (within established tolerances) is shown in Fig. 18. In ascertaining the fact of informational perfection, among others one can note the remarkable result concerning informational optimality.

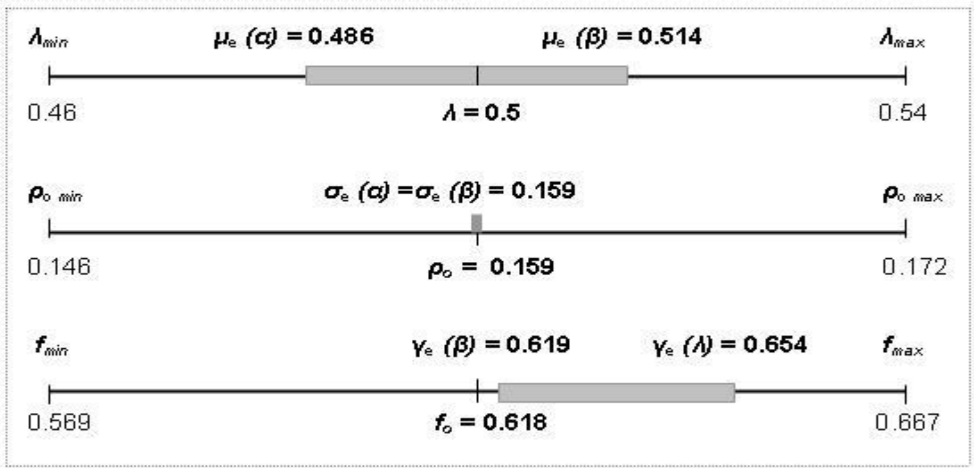


Fig. 18: The positioning of normalized statistical parameters of the system of melting and boiling temperatures of the Elements within PNR' permissible ranges

## Appendix 6: Manifestations of blood dimensional perfection

Biomedical data, e.g. lab-tests, are of special value and interest as objects of the analysis and information identification with such conceptions of dimensional perfection as informational optimality, mathematical harmony and balance, because hypothetically respective norms intrinsic to healthy human as the results of natural evolution and of researches progressing tend to the perfection. Negative outcomes of the data norms identification might give rise the need of improvement in specifying the norms or methods of the data estimation and/or their interpretation.

When applying to various manifestations of dimensional perfection of blood parameters we will deal with general physical ones as volume, temperature, pressure, the clotting, pulse rate that are the more expected in the perfection realization. The expectation is based on the obvious assumption that these characteristics in more extent originate from biological evolution, and less from progressing the biomedical practice and different approaches in establishing normal values.

*Blood volume*

The adult human body is known to contain approximately 5 liters of blood making up 7 to 8% of a person's body weight, and approximately 2.75 to 3 liters of blood is plasma and the rest is the cellular portion. Thus the volume or mass ratio of human blood and human body does not exceed permissible range (8%) for $\rho_o$, i.e. these characteristics are informatively optimized. The volume ratio of averaged blood plasma (2.875 liters) and blood as a whole (5 liters) equals 0.575. This result differs from $f_o$ on approximately 7% and thus may be qualified as the manifestation of dimensional harmony.

*Blood temperature*

The informational optimality, harmony and balance of blood temperature were comprehensively demonstrated in the recent investigation when using the common temperature scale of blood and water [37]. The additional illustration adequate to the present discussion consists in the following. One of obtained results amounts to the optimality of the approximate range 44.4 to 36.8°C for normal blood temperature of warm-blooded on the temperature scale, namely the ratio (44.4 – 36.8)/44.4 = 0.171 that on 7.7% differs from $\rho_o$ = 0.159, i.e. this result is within permissible for the optimality range. Another notable illustration of perfection amounts to the human temperature variation effects. When comparing the critical for human life body temperatures: 28°C (severe heart rhythm disturbances are likely and breathing may stop at any time) and 43°C (cardio-respiratory collapse will occur), their ratio = 0.651 only on 5.4% differs from $f_o$ that is to be accepted as the manifestation of harmonious relation.

*Blood pressure*

It is noted the human's cardiac performance is subordinated to the golden section law that is expressed in the observation that the ratio of the minimum (diastolic) pressure to the maximum (systolic) pressure is equal, on the average, to 0.625, that is, it is close to the golden ratio. It is assumed this coincidence reflects some objective regularity of the cardiac activity harmonic organization. We will widen this observation by analyzing various ratios of normal diastolic and systolic values currently accepted in USA and UK medical practice [39]. The results of the analysis are presented in Table 14.

It can be seen the harmony between determining normal values of systolic and diastolic pressure exists in full in UK system of estimating normal blood pressure and is very close to the harmony for the USA system. This indicates onto the difference in the approach to establishing the normal values in these two systems having an influence upon the recognition of dimensional perfection.

Table 14: Normal systolic and diastolic blood pressure specified in USA and UK

| Country | Blood pressure type | Normal values (mmHg) | $D_{min}/S_{min}$ | $D_{max}/S_{max}$ | $C_{inf}$ | Deviation from $C_{inf}$ (%) |
|---|---|---|---|---|---|---|
| USA | Systolic (S)<br>Diastolic (D) | 120 – 140<br>80 – 95 | 0.667 | 0.679 | $f_o$<br>$f_o$ | +7.9<br>+9.8 |
| UK | Systolic (S)<br>Diastolic (D) | 110 – 140<br>70 – 90 | 0.636 | 0.643 | $f_o$<br>$f_o$ | +2.9<br>+4.0 |

*Pulse rate*

Referring to blood flow, we can supplement the consideration of human heartbeat by analyzing human's pulse rate ratios. The data taken from [40] and the results of their analyzing are presented in Table 15.

Table 15: Normal pulse ($N$) in beats per minute for a resting heart

| Category | Bottom ($N_B$) | Top ($N_T$) | Ratio $N_B/N_T$ | Identification to $C_{inf}$ | Deviation from $C_{inf}$ (%) |
|---|---|---|---|---|---|
| Newborn infants | 100 | 160 | 0.625 | $f_o$=0.618 | +1.1 |
| Children 1 to 10 years | 70 | 120 | 0.583 | $f_o$=0.618 | –5.7 |
| Children over 10 and adults | 60 | 100 | 0.600 | $f_o$=0.618 | –2.9 |
| Well-trained athletes | 40 | 60 | 0.667 | $f_o$=0.618 | +7.9 |

Table 15 undoubtedly demonstrates the location of pulse rate ratios within the permissible range of $f_o$ (from 0.569 to 0.667). It is also noteworthy the mean of the ratios, i.e. $(\Sigma N_B/N_T)/5 = 0.619$ is very close to $f_o = 0.618$ that additionally illustrates the phenomenon of harmony in this case.

*Blood clotting*

The normal values for platelets important for proper blood clotting specified for general estimation and the estimation in blood test practice [41] are presented in Table 16 that illustrates the conformity of normal values of platelets with dimensional balance and harmony.

Table 16: Normal values of platelets in cubic millimeter of blood

| Estimation category | Bottom ($N_B$) | Top ($N_T$) | $1 - N_B/N_T$ | Identification to $C_{inf}$ | Deviation from $C_{inf}$ (%) |
|---|---|---|---|---|---|
| General estimation | 250,000 | 500,000 | 0.5 | $\lambda = 0.5$ | 0 |
| Blood test practice | 150,000 | 450,000 | 0.667 | $f_o = 0.618$ | 7.9 |

*Blood biochemical composition*

The highly representative manifestation of dimensional perfection of normal and critical test values, established in medical practice, relates to such important blood tests' as the content of glucose and cholesterol in blood plasma. Normal values of glucose in blood are ranging between 70 and 115 mg/dL, and therefore the ratio 70/115 = 0.609 differs from $f_o$ = 0.618 only on 1.5% that proves the harmony of the values. As for cholesterol, risk factors for heart disease occurs when total cholesterol is between 200 to 239 mg/dL, and thus the ratio (239 – 200)/239 = 0.163 differs from $\rho_o$ = 0.159 on 2.5% representing dimensional optimality. Another kind of perfection is connected to the ratio of bad cholesterol (LDL) to good cholesterol (HDL). Risk levels for heart attack or stroke due to blocked arteries: an average risk: 4.4 to 7.1; a moderate risk: 7.1 to 11.0. Thus, respective ratios 4.4/7.1 = 0.620 and 7.1/11.0 = 0.645 illustrates the harmony of these values with errors 1% and 4.4% respectively.

The data of comparing normal ranges with information constants of the rest of blood tests borrowed from [41] are presented in Table 17. These data demonstrate wide range (from 0% to almost 26%) of deviations from information constants.

Table 17: Blood tests' normal values and data of the comparison with information constants

| # | Name of blood test | Normal range $B \div A$ | $B_j/A_j$<br>$1 - B_j/A_j$ | Identification to $C_{inf}$ | Deviation from $C_{inf}$ (%) |
|---|---|---|---|---|---|
| 1 | BUN (Blood urea nitrogen) mg/dL | 11 – 23 | 0.478 | $\lambda$=0.5 | –4.3 |
| 2 | Albumin (in g/dL) | 3.50 – 5.50 | 0.636 | $f_o$=0.618 | +2.9 |
| 3 | Aldolase (in U/L) | 1.7 – 4.9 | 0.653 | $f_o$=0.618 | +5.7 |
| 4 | Aldosterone (in ng/dL) | 5 – 30 | 0.167 | $\rho_o$=0.159 | +7.9 |
| 5 | AAT ($Alpha_1$-Antitrypsin) mg/dL | 126 – 225 | 0.56 | $f_o$=0.618 | –9.4 |
| 6 | Amilase (U/L) | 35 – 115 | 0.696 | $f_o$=0.618 | +12.6 |
| 7 | Ammonia (in µM/L) | 10 – 50 | 0.2 | $\rho_o$=0.159 | +25.8 |
| 8 | Antithrombin lll in mg/dL | 17 – 30 | 0.567 | $f_o$=0.618 | –8.2 |
| 9 | Apoliphrotein "A" (male) mg/dL | 90 – 155 | 0.581 | $f_o$=0.618 | –6.0 |
| 10 | Apoliphrotein "A" (female) mg/dL | 94 – 172 | 0.546 | $\lambda$=0.5 | +9.2 |
| 11 | Apoliphrotein "B" (male) mg/dL | 55 – 100 | 0.55 | $\lambda$=0.5 | +10 |
| 12 | Apoliphrotein "B" (female) mg/dL | 45 – 110 | 0.591 | $f_o$=0.618 | –4.4 |
| 13 | AST Aspartate aminotransferase (U/L) | 14 – 50 | 0.72 | $f_o$=0.618 | +16.5 |
| 14 | Bilirubin (in mg/dL) | 0.3 – 1.1 | 0.727 | $f_o$=0.618 | +17.6 |
| 15 | Vitamin A (in mg/dL) | 20 – 50 | 0.6 | $f_o$=0.618 | –2.9 |
| 16 | Vitamin $B_1$ (in mg/dL) | 3 – 7 | 0.571 | $f_o$=0.618 | –7.6 |
| 17 | Vitamin $B_2$ (in mg/dL) | 3.7 – 13.7 | 0.73 | $f_o$=0.618 | +18. |
| 18 | Vitamin $B_6$ (in ng/mL) | 5 – 30 | 0.167 | $\rho_o$=0.159 | +7.9 |
| 19 | Vitamin $B_{12}$ (in pg/mL) | 150 – 750 | 0.73 | $f_o$=0.618 | +18.1 |
| 20 | Vitamin C (in mg/dL) | 0.4 – 1.5 | 0.733 | $f_o$=0.618 | +18.6 |

| | | | | | |
|---|---|---|---|---|---|
| 21 | Vitamin E (in µg/mL) | 5.5 – 17 | 0.676 | $f_o$=0.618 | +9.4 |
| 22 | Vitamin K (in ng/mL) | 0.2 – 1 | 0.2 | $\rho_o$=0.159 | +25.8 |
| 23 | GGT (U/L) | 5 – 54 | 0.585 | $f_o$=0.618 | –5.3 |
| 24 | Haptoglobin (in mg/dL) | 40 – 270 | 0.148 | $\rho_o$=0.159 | –6.9 |
| 25 | HCT – hematocrit (%) | 42 – 52 | 0.192 | $\rho_o$=0.159 | +20.7 |
| 26 | Hb, hemoglobin (male) g/dL | 16 – 18 | 0.698 | $f_o$=0.618 | +13 |
| 27 | Hb, hemoglobin (female) g/dL | 12 – 16 | 0.75 | $f_o$=0.618 | +21.4 |
| 28 | Globulin (in g/dL) | 2 – 3.5 | 0.571 | $f_o$=0.618 | –7.6 |
| 29 | K (Kalium, Potassium) mEq/L | 3.5 – 5.3 | 0.660 | $f_o$=0.618 | +6.8 |
| 30 | Ca (Calcium) in mg/dL | 8.9 – 10.7 | 0.168 | $\rho_o$=0.159 | +5.7 |
| 31 | Caroten (in mg/dL) | 48 – 200 | 0.76 | $f_o$=0.618 | +23 |
| 32 | Creatinine clearance in mg/dL | 0.8 – 1.2 | 0.667 | $f_o$=0.618 | +7.9 |
| 33 | Complement (U) | 25 – 110 | 0.773 | $f_o$=0.618 | +25.1 |
| 34 | Cortisol (in mg/dL) | 6 – 23 | 0.739 | $f_o$=0.618 | +9.4 |
| 35 | Creatinine (in mg/dL) | 0.8 – 1.2 | 0.667 | $f_o$=0.618 | +7.9 |
| 36 | Lactate (in mg/dL) | 5 – 20 | 0.75 | $f_o$=0.618 | +21.4 |
| 37 | LDH (Lactate dehydrogenase) U/L | 100 – 250 | 0.6 | $f_o$=0.618 | –2.9 |
| 38 | WBC (Leukocytes) (1/ µL) | 4600-10200 | 0.451 | $\lambda$=0.5 | –9.8 |
| 39 | Lymphocytes (%) | 20 – 40 | 0.5 | $\lambda$=0.5 | 0 |
| 40 | Mg (Magnesium) in mg/dL | 1.7 – 2.2 | 0.773 | $f_o$=0.618 | +25.1 |
| 41 | Copper (in mg/dL) | 70 – 150 | 0.47 | $\lambda$=0.5 | –6.0 |
| 42 | Monocytes (%) | 2 – 8 | 0.75 | $f_o$=0.618 | +21.4 |
| 43 | Uric acid (in mg/dL) | 3.5 – 8.5 | 0.588 | $f_o$=0.618 | –4.8 |
| 44 | Urea (in mg/dL) | 24 – 49 | 0.49 | $\lambda$=0.5 | –2.0 |
| 45 | Na (natrium) in mmol/L | 137 – 145 | 0.654 | $f_o$=0.618 | +5.8 |
| 46 | Neutrophils (%) | 40 – 60 | 0.667 | $f_o$=0.618 | +7.9 |
| 47 | PRA (Renin) in ng/ml/h | 1.9-3.7 | 0.513 | $\lambda$=0.5 | +2.6 |
| 48 | PSA (in ng/mL) | 4 – 6.5 | 0.615 | $f_o$=0.618 | –0.5 |
| 49 | MCH (Mean Cell Hemoglobin) in pg/Rbc | 27 – 31 | 0.129 | $\rho_o$=0.159 | –18.9 |
| 50 | MPV (Mean platelet volume) in fL | 7.4 – 10.4 | 0.711 | $f_o$=0.618 | +15.0 |
| 51 | MCHC (ng/dL) | 30 – 36 | 0.167 | $\rho_o$=0.159 | +5.0 |
| 52 | Testosterone (male) in ng/dL | 300 – 1200 | 0.75 | $f_o$=0.618 | +21.4 |
| 53 | Testosterone (female) ng/dL | 20 – 75 | 0.733 | $f_o$=0.618 | +18.6 |
| 54 | TSH (mU/L) | 0.4 – 4.8 | 0.167 | $\rho_o$=0.159 | +5.0 |
| 55 | $T_4$ – free thyroxine in ng/dL | 0.9 – 2.0 | 0.45 | $\lambda$=0.5 | –10 |
| 56 | $T_4$ – total thyroxine in ng/dL | 4.8 – 11.2 | 0.571 | $f_o$=0.618 | –7.6 |
| 57 | $T_3$ –triiodothyronine inng/dL | 70 – 190 | 0.632 | $f_o$=0.618 | +2.3 |
| 58 | $RT_3U$ (%) | 35 – 38 | 0.497 | $\lambda$=0.5 | –0.6 |
| 59 | Platelets (in $10^3$/mL) | 150 –450 | 0.667 | $f_o$=0.618 | +7.9 |
| 60 | Ferritin (in ng/mL) | 20 – 300 | 0.579 | $f_o$=0.618 | –6.3 |
| 61 | Fibrinogen (in mg/dL) | 200 – 400 | 0.5 | $\lambda$=0.5 | 0 |
| 62 | Folic Acid (in ng/ml) | 3.5 – 17 | 0.206 | $\rho_o$=0.159 | +29.6 |
| 63 | FSH (for female) in mU/mL | 4 – 30 | 0.133 | $\rho_o$=0.159 | –16.3 |
| 64 | FSH (for male) in mU/mL | 4 – 25 | 0.16 | $\rho_o$=0.159 | +0.6 |
| 65 | Phosphorus (in mg/dL) | 2.5 – 4.5 | 0.555 | $\lambda$=0.5 | +11 |
| 66 | Chloride (in mmol/L) | 98 – 107 | 0.528 | $\lambda$=0.5 | +5.6 |
| 67 | ALP (Alkaline phosphatase) U/L | 35 – 150 | 0.767 | $f_o$=0.618 | +24.1 |
| 68 | Eosinophils: neu-neutrophils (%) | 40 – 60 | 0.667 | $f_o$=0.618 | +7.9 |
| 69 | Eosinophils: lym-lymphocites (%) | 20 – 40 | 0.5 | $\lambda$=0.5 | 0 |
| 70 | Eosinophils: mono-monocytes (%) | 2 – 8 | 0.75 | $f_o$=0.618 | +21.4 |
| 71 | Eosinophils: eos- eosinophils (%) | 1 – 4 | 0.75 | $f_o$=0.618 | +21.4 |
| 72 | RBC in million/µL | 4.2 – 6.2 | 0.667 | $f_o$=0.618 | +7.9 |
| 73 | $E_2$, Estradiol (female) in pg/ml | 30 – 400 | 0.472 | $\lambda$=0.5 | –5.6 |
| 74 | $E_2$, Estradiol (male) in pg/ml | 10 – 50 | 0.2 | $\rho_o$=0.159 | +25.8 |

*Criteria and results of multi-component analysis*

Since physical and biomedical parameters of blood represent the multi-component system, its analysis over a degree of dimensional perfection of the system as a whole requires special criteria. In the assumption of uniformly distributed ratios *B/A*, the natural confirming percentage (which is by no means a rate of system's perfection) is determined as follows:

$$P_d = \rho_o(\rho_o + f_o + \lambda)100\% \approx 20\% \quad (51)$$

The exceeding of total conforming percentage $P_t$ above $P_d$ is the tendency to dimensional perfection. In reality the weighed percentage $P_{tw}$ is to be used instead of $P_t$ to this purpose that is calculated as follows:

$$P_{tw} = (n_\rho P_\rho + n_f P_f + n_\lambda P_\lambda)/( n_\rho + n_f + n_\lambda), \quad (52)$$

where $P_\rho$, $P_f$, and $P_\lambda$ = the conforming percentage in regard of $\rho_o$, $f_o$, and $\lambda$,
$n_\rho$, $n_f$, $n_\lambda$ = the number of the conforming in regard of $\rho_o$, $f_o$, and $\lambda$.

Clearly, the permissible for the estimation range of $P_{tw}$ is $P_d \leq P_{tw} \leq 100\%$. When $P_{tw} > P_d$, certain ratio over the condition $1 \leq P_{tw}/P_d \leq 5$ can serve for deciding whether an analyzed system possesses enough dimensional perfection. This ratio is useful to consider as the argument of positive logarithmic function $Q = \ln(P_{tw}/P_d)$ representing the measure of such decision quality that meets the following requirements:

when $P_{tw}/P_d = 1$, $Q_{min} = 0$;
when $P_{tw} = 100\%$, $Q_{max} = 1.61$;
when $P_{tw}/P_d = e$ (Napier's constant), $Q_o = 1$

It is reasonably to consider $Q_o = 1$ as being sufficient to qualify an analyzed system as having been located in the area of dimensional perfection. Thus, the following criterion for recognizing a system possessing such quality is suitable for practical estimations:

$$P_{tw}/P_d \geq 2.7 \quad \text{or} \quad P_{tw} \geq 55\% \quad (53)$$

Clearly, this means the area of dimensional perfection is $55\% \leq P_{tw} \leq 100\%$, and when $P_{tw} = 100\%$, a system is being qualified as completely perfect. Interestingly the existence of perfection in the criterion itself, namely: (a) for the quality function the ratio $Q_o/Q_{max} = 0.62$ quite precisely represents mathematical harmony, and (b) for the ratios $P_{tw}/P_d$: (2.7 – 1)/2.7 = 0.63 - the harmony, and (5 – 2.7)/5 = 0.54 - the balance.

For analyzing blood parameters and tests as a multi-component system the percentage of cases conforming with dimensional perfection, and numbers of the conforming cases, resulting from statistical analysis of all data, presented in previous section and Table 17, are summarized in Table 18.

Table 18: Data of analyzing blood parameters and tests as a multi-component system

| Information constant | Number of cases conforming with dimensional perfection | Percentage of cases conforming with dimensional perfection |
|---|---|---|
| $\lambda$ | $n_\lambda = 13$ | $P_\lambda = 72$ |
| $f_o$ | $n_f = 39$ | $P_f = 64$ |
| $\rho_o$ | $n_\rho = 10$ | $P_\rho = 59$ |

Basing on these data and by using the expressions (53), the following calculation result is true when estimating to what extent the system of blood parameters and tests conforms (if any) to the dimensional perfection:

$$P_{tw} = (n_\rho P_\rho + n_f P_f + n_\lambda P_\lambda)/( n_\rho + n_f + n_\lambda) =$$
$$= (10x59 + 39x64 + 13x72)/(10 + 39 + 13) = 64.9\% \qquad (54)$$

In so far as this value exceeds 55%, according to the criterion (49) the system of blood parameters and tests may be qualified as being located in the area of dimensional perfection.

## Appendix 7: Quality loss for various diagrams of weights

Expression (9) needs to be analyzed itself for investigating the limiting values regarding theoretically boundary diagrams of weights among their infinite variety that might be occurring in a real practice. Some expectable averaged diagram is reasonable to define as well. Three models of diagrams, presented in Table 19, are being appropriate to this purpose.

Table 19: Expression of $L_q$ for three types of weight diagrams

| # | Type of weights' diagram | Modeling formula | $L_q$ expression |
|---|---|---|---|
| 1 | Max. convex diagram | $(N-1)\, K_{max} + \rho_o K_{max} = 1$ | $L_{q1} = \rho_o / N$ |
| 2 | Max. concave diagram | $K_{max} + (N-1)\, \rho_o K_{max} = 1$ | $L_{q2} = \rho_o\, (N-1)/N$ |
| 3 | Linear diagram | $0.5\, K_{max}\, (N+1) = 1$ | $L_{q3} = 0.5\, \rho_o\, (\rho_o + 1/N)$ |

Calculation results over Table 19 expressions are graphically illustrated in Fig.19. according to which the following can be stated:

(a) Maximal quality losses can never exceed the theoretical limit of $L_{q2}$, i.e. $\rho_o = 1/2\pi = 0.159$.

(b) In terms of quality losses the linear diagram of weights holds a prominent place as being some intermediate between two other boundary diagrams. This quality enables to consider the linear diagram as an appropriate model to a variety of estimations. Within very wide range of $N$ the quality losses $L_{q3}$ are from 0.013 to 0.05.

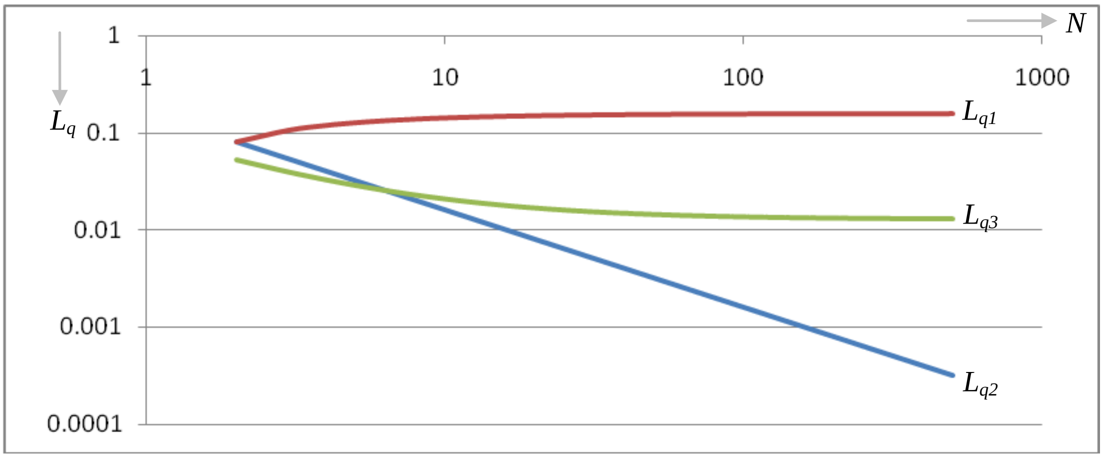


Fig.19: Graphical illustration of quality losses for three types of weight diagrams

According to Fig. 19 the following can be stated:

(a) Maximal quality losses can never exceed the theoretical limit of $L_{q2}$, i.e. $\rho_o = 1/2\pi = 0.159$.

(b) In terms of quality losses the linear diagram of weights holds a prominent place as being some intermediate between two other boundary diagrams. This quality enables to consider the linear diagram as an appropriate model to a variety of estimations. Within very wide range of $N$ the quality losses $L_{q3}$ are from 0.013 to 0.05.

## Appendix 8: Entropy method for proving 7±2 phenomenon

The considerable attention is deserved to the 7 as the often occurring classification integer. In the nature this bears upon seven periods of the periodic table of elements, seven colors of the rainbow, the number of notes, seven levels of taxonomic classification, etc. In human activity one may note seven lamps of architecture [42], seven "Liberal Arts" aimed at imparting general knowledge and developing intellectual capacities [43], and many others. Classification character of the number seven (completeness, fullness, or perfection as fundamental symbolic meanings in Scripture) is of significance in religion too [44]. In famous experiments on humans' capacity for transmitting information George A. Miller made the important step in comprehending the magical 7 [45]. He proved the interval between 5 and 9 equally-weighted error-less choices characterizes people's memory performance on random lists of letters, words, and numbers. Miller hypothesized that the obtained average result 7 along with a lot of magical manifestations of lucky number seven in science, technology and other fields might represent either 1) something deep and profound or 2) is just a pernicious, Pythagorean coincidence. When using conceptions of informational optimality, harmony and balance and the entropy approach to classification integers, one can prove the first of these hypotheses is true.

Applying information theory, one may treat classification integers as well as the Miller's 7±2 phenomenon in terms of PNRs regarding ratios $1/H_x$, where $H_x = - ln\ (1/x) = ln\ (x)$ is the entropy of $x$ classification situations representing equally-weighted error-less choices. Now, referring to permissible deviations $f_o \pm f_o\ /4\pi$ and $\lambda \pm \lambda\ /4\pi$, one may proceed to the analysis aimed at revealing for which arguments $R\{x\}$ and to what extent the entropy ratio $ln\ (x_{min})/ln\ (x) = 1/ln\ (x)$ meets the requirements of harmony and balance for the six ($i$ = 1÷ 6) critical values: 1) $f_{o1} = f_o = 0.618$, 2) $f_{o2} = f_o + f_o\ /4\pi = 0.667$, 3) $f_{o3} = f_o - f_o\ /4\pi = 0.569$; 4) $\lambda_4 = \lambda = 0.5$, 5) $\lambda_5 = \lambda + \lambda\ /4\pi = 0.54$, and 6) $\lambda_6 = \lambda - \lambda\ /4\pi = 0.46$.

The calculations, results of which are presented in Fig. 20, have been performed using both the entropy ratio $1/ln\ (x)$ (see diagram D1) and the absolute values of differences between the critical values ($f_{oi}$, $\lambda_i$) and this ratio (see diagram D2).

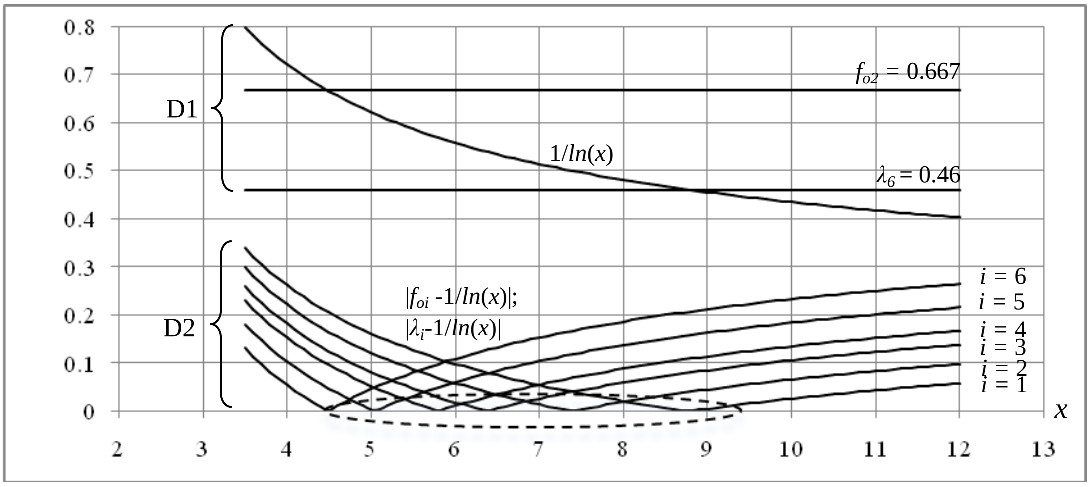


Fig. 20: Graphical illustration of entropy perfection regarding 7±2

Regarding the entropies ratios, only integers from 5 to 9 (as rounding off values) are within permissible limits bounding the harmony and balance (D1). The minimums of all calculated differences $|f_{oi} - 1/ln\ (x)|$ and $|\lambda_i - 1/ln\ (x)|$ are appeared (D2) located within the range from 4.5 to 9.4 on the axis $x$ (dashed line on Fig. 20), within which, if rounding off, the classification integers meeting the requirement of harmony and balance are located between 5 and 9 inclusive. This is an evident proof for the Miller's 7±2 phenomenon.

In a very broad sense there are two kinds of classification: 1) a quantitative, e.g. periodic table of elements, and 2) a qualitative, e.g. taxonomic classification. The closeness of classification systems to perfection, including those arising owing to the needs and for a regulation of human practical activity, apparently, depends on evolution laws. The expected relative imperfection of quantitative and qualitative gradations as a whole in human practical activity is arising because of objective reasons, such as technological and/or economic ones as, for instance, in accuracy classification in industry, or due to subjective ones dependent on the developers and of used methods intrinsic to classification systems in various evaluations.

## Appendix 9: Universal accuracy classification scale

The optimum characteristics of accuracy hierarchy have enabled to develop the *Universal Accuracy Classification Scale* (UACS), which consists of the set of dimensionless base-line numerical values plotted as system elements, subclasses, classes, and system groups that is presented in Table 20. The tables involve the module of relative numbers for a group of classes, which with multiplying factors ($F_m$) can be transformed into the classification system of any degree of complexity. Each of existing in practice accuracy classification systems might be identified in terms of UACS.

Table 20: Universal Accuracy Classification Scale

| Class and Multiplying factor $C[F_m]$ | | | | Relative classification elements ($e_i$) & subclasses ($E_i$) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ | |
| | | | | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ |
| ……… | ……… | ……… | ……… | ……… | ……… | ……… | ……… | ……… | ……… | ……… |
| ……… | ……… | ……… | ……… | ……… | ……… | ……… | ……… | ……… | ……… | ……… |
| $C_6[10^{-5}]$ | $C_{12}[10^{-11}]$ | $C_{18}[10^{-17}]$ | … | 10 | 8.8 | 7.4 | 5.9 | 4.5 | 3.1 | 1.6 |
| $C_5[10^{-4}]$ | $C_{11}[10^{-10}]$ | $C_{17}[10^{-16}]$ | … | 6.4 | 5.5 | 4.6 | 3.7 | 2.8 | 1.9 | 1.0 |
| $C_4[10^{-3}]$ | $C_{10}[10^{-9}]$ | $C_{16}[10^{-15}]$ | … | 4.0 | 3.5 | 2.9 | 2.3 | 1.8 | 1.2 | 0.64 |
| $C_3[10^{-2}]$ | $C_9[10^{-8}]$ | $C_{15}[10^{-14}]$ | … | 2.5 | 2.2 | 1.8 | 1.5 | 1.1 | 0.76 | 0.40 |
| $C_2[10^{-1}]$ | $C_8[10^{-7}]$ | $C_{14}[10^{-13}]$ | … | 1.6 | 1.4 | 1.1 | 0.92 | 0.70 | 0.48 | 0.25 |
| $C_1[10^{0}]$ | $C_7[10^{-6}]$ | $C_{13}[10^{-12}]$ | … | 1.0 | 0.86 | 0.72 | 0.58 | 0.44 | 0.30 | 0.16 |

The 18 classes are placed in the table just illustratively. Formally, the UACS might be represented as the infinite number of hierarchy levels. However, application fields impose restrictions. In metrology the fundamental physical restriction to measurement exists that confines the levels of such a scale in principle. The restriction is known in physics as *Heisenberg uncertainty*. This smallest uncertainty has also connection with information cyclicity and limits the scale of accuracy classification by the levels (◊).

◊ Note. *There is the following intuitive assumption that arises from the existence of uncertainty limit: the informational cyclicity and Heisenberg's uncertainty make possible to conceive universe information in terms of metrology. Any system can be controllable by the condition of appropriate accuracy; otherwise, the lack of measurement information necessary for a stable control may cause system's collapse with the probability being increased with approaching to the highest hierarchical level of accuracy classification. Possibly there is a kind of balance between thermodynamic and information entropies and, speaking figuratively, between alive and lifeless nature. The supposition is that the accumulation of information caused by a life activity may become critically uncertain for ensuring the stable control of measurement information and can lead to the big explosion. The cycles of collecting information and increasing the thermodynamic entropy infinitely recur and, possibly, this infinite balancing process precludes the possibility of the thermal death of the universe. Largely, this scenario supplements the so-called big-bang theory with information approach. Besides, if local explosion and contraction cycles are possible, they may cause the existence of a number of univers parts where information is collected. All that may dramatically change a notion of the origin of life, namely, instead of being either unique or accidental, it becomes a natural phenomenon. In a sense, the penetrating assumption of Wicken that the evolution and the origin of life are not separate problems [46] is of principle nature.*

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